IJEIT
The International Journal of Engineering and Information Technology
journal homepage:www.ijeit.misuratau.edu.ly
جامعة مصراتة
MISURATA UNIVERSITY

The Effective Method of Solving Unit Commitment Problems Based on The Lagrange Relaxation Technique

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Abstract-- **This paper presents the Unit Commitment (UC) and the common constraints that could appear while doing the UC. Also, Dynamic Programming is introduced briefly with its advantages and disadvantages. Lagrange Relaxation (LR) is introduced with its advantages and disadvantage. Finally, a numerical example of the IEEE model with 36 units and 118 buses are used for unit commitment scheduling within 24 hours.**

Index Terms: **Unit Commitment, dynamic Programming, Constraints, Spinning Reserve, Lagrange Relaxation, Lagrange Multipliers.**

I. INTRODUCTION

The task of Unit Commitment (UC) involves scheduling the on/off status, as well as the real power outputs, of thermal units for use in meeting forecasted demand over a future short-term (24–168 hour) horizon. The resultant schedule should minimize the system production cost during the period while simultaneously satisfying the load demand, spinning reserve, physical and operational constraints of the individual unit. Since improved UC schedule may save the electric utilities millions of dollars per year in production costs, UC is an important optimization task in the daily operation planning of modern power systems [1].

Over the past thirty-five years, Lagrangian relaxation (LR) has become one of the most practical and accepted approaches to solve for real-sized unit commitment (UC) problems. LR has also been used in other power engineering applications, such as hydro-thermal and other scheduling problems in deregulated systems. The key idea in LR-based approaches is to append system-wide power and reserve balance constraints to the objective function with their corresponding dual variables. The dual to the Lagrangian is found to have a separable structure [2].

Received 13 Apr, 2020; revised 4 May, 2020; accepted 14 May, 2020.

Available online 15 May, 2020.

II. UNIT COMMITMENT (UC)

Varying the loads along the day time and the weekdays requires the operators to manage their generation in order to supply the loads economically. The total load on the system will generally be higher during the daytime and early evening when industrial loads are high, lights are on, and so forth, and lower during the late evening and early morning when most of the population is asleep. In addition, the use of electric power has a weekly cycle, the load being lower over weekend days than on weekdays. Therefore, rather than committing enough units (turning them on) to supply the maximum demand and keeping them online all the time. A great deal of money can be saved by turning units off (de-committing them) when they are not needed [3].

Therefore, Unit commitment is one of the critical issues in the economic operation of a power system. It determines unit generation schedule for minimizing the operating cost and satisfying the prevailing constraints such as load balance, system spinning reserve, ramp-rate limits, fuel constraints, as well as a minimum up and downtime limits over a set of time periods. With the unit commitment schedule, generating companies satisfy customer load demands and maintain transmission flows within their permissible limits [3].

III. CONSTRAINTS IN UNIT COMMITMENT

Any optimum unit commitment procedure must produce a schedule that can be implemented in a real-life system taking into account a large number of practical systems, devices, operational and environmental considerations; generator equality, and inequality.

Equality constraints mainly described by the power balance equation, also it's known as demand constraint [4].

$$\sum_{i=1}^{N} P_i = P_D \qquad (1)$$

Where P_D is the load demand and N is the number of units committed at a particular hour.

Inequality constraints are:

- **Thermal Unit Constraints:**
  Thermal units usually require a crew to operate them, especially when turned on and turned off. A thermal unit can undergo only gradual temperature changes, and this translates into a time period of some hours required to bring the unit on-line. As a result of such restrictions in the operation of a thermal plant, various constraints arise, such as [5]:
  1. **Minimum uptime:** once the unit is running, it should not be turned off immediately.
  2. **Minimum downtime:** once the unit is de-committed, there is a minimum time before it can be recommitted.
  3. **Crew constraints:** The number of units that can be started at the same time in a particular plant depends upon the limited personal (crew) available.

- **Unit generation capability limits:**
  The upper and lower limits of generation of the units force them to operate within these boundaries of operation [4].
- **Ramp Rate Constraint:**
  The rate of increasing or decreasing electrical output from the unit is restricted by the ramp rate limit [4].
- **Transmission Flow Constraints:**
  For power supplies that have to utilize heavily loaded lines and transformers located far from loads, transmission flow limits throughout the system may become troublesome [3].
- **Must Run**
  Some units are given a must-run status during certain times of the year for the reason of voltage support on the transmission network or for such purposes as a supply of steam for uses outside the steam plant itself [5].
- **Spinning Reserve**
  Spinning reserve is the term used to describe the total amount of generation available from all units synchronized on the system, minus the present load and losses being supplied. The spinning reserve must be carried so that the loss of one or more units does not cause too far a drop-in system frequency. Quite simply, if one unit is lost, there must be ample reserve on the other units to make up for the loss in a specified time period [5].
- **Fuel Constraints**
  The fuel supply constraints experienced by utilities are varied. Many utilities have gas supplies that are limited or which have taken-or-pay requirements. Other fuels can be limited due to supply problems, limited storage facilities or other reasons. Often, fuels will be constrained over a period much longer than the one-week time horizon of the unit commitment problem [6].

IV. DYNAMIC-PROGRAMMING SOLUTION

Dynamic programming (DP) has many advantages over the priority list and enumeration techniques. DP searches the solution space that consists of the units' status for an optimal solution. The search can be carried out in a forward or backward direction. The time period of the study horizon is known as the states [4]. The advantage of DP is the ability to maintain solution feasibility. DP builds and evaluates the complete "decision tree: to optimize the problem at hand, thus, it sufQWfers from the course of dimensionality because the problem size increases rapidly with the number of generation units to be committed, which results in unacceptable solution time. To reduce the dimension, search space and execution time, several approaches have been developed. The most extensively used approach is the priority list technique [4].

Suppose we have four units in a system and any combination of them could serve the (single) load. There would be a maximum of 2^4 -1 = 15 combinations to test. However, if a strict priority order is imposed, there are only four combinations to try [5]:
Priority 1 unit
Priority 1 unit + Priority 2 unit
Priority 1 unit + Priority 2 unit + Priority 3 unit
Priority 1 unit + Priority 2 unit + Priority 3 unit + Priority 4 unit

- **Forward Dynamic Problem Approach:**
  The forward approach has noticeable advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of the time it has been off-line, then a forward dynamic-program approach is more suitable since the previous history of the unit can be computed at each stage. There are other practical reasons for going forward. The initial conditions are easily specified and the computations can go forward in time as long as required [5].
- **Problem Formulation**
  The algorithm to compute the minimum cost in hour K with a combination I is as the following:

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1,L:K,I) + F_{cost}(K-1,L)] \qquad (2)$$

Where
$F_{cost}(K,I)$ = least total cost to arrive at state (K, I)

$P_{cost}(K,I)$ = production cost for state (K-1, L) to state (K, I)

$S_{cost}(K-1,L:K,I)$ = transition cost from state (K-1, L) to state (K, I)

The flowchart below shows how this algorithm works:

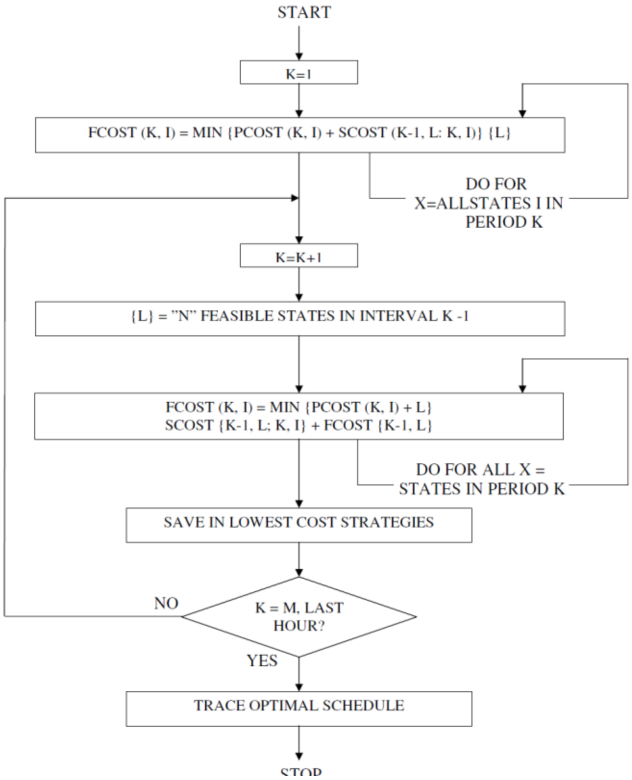

Figure 1. Basic Configuration of the FDP Algorithm

State (K, I) is the I_{th} combination in hour K.
X = number of states to search each period.
N = number of strategies, or paths, to save at each step.

The main disadvantage of the DP is the difficulty of solving directly the problem when numerous variables representing operating conditions are involved.

V. LAGRANGE RELAXATION (LR) METHOD

The approach decomposes the unit commitment problem into a master problem and makes better and easily manageable sub-problems to be solved separately. The sub-problems are linked by Lagrange multipliers that are added to the master problem to yield a dual problem. The dual problem has lower dimensions than the primal problem and easier to solve [4].

The Lagrange multipliers are computed at the master problem level. Once computed, multipliers are passed to the sub-problems. The solution of the sub-problem is feedback to the master problem and updated multipliers are obtained and used by the sub-problems again. This process is repeated until the solution converges. For short term UCP, the multipliers are updated through a sub-gradient method. For long term UCP, multipliers are updated with the variable metric method to prevent the solution near the dual maximum [4].

The Lagrange relaxation technique has emerged as an effective method of solving the unit commitment problem. Compared with approaches, Lagrange relaxation is more flexible for handling different types of operating constraints in a power system and has higher computational efficiency.

The objective functions several constraints and of the unit commitment problem [5]:

1. Loading constraints:

$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t = 0 \quad \text{for t = 1,…....,T} \qquad (3)$$

2. Unit limits:

$$U_i^t P_i^{\min} \le P_i^t \le U_i^t P_i^{\max} \quad \text{for i= 1,…, N and t = 1,...,T} \qquad (4)$$

3. The objective function is:

$$\sum_{t=1}^{T}\sum_{i=1}^{N}[F_i(P_i^t) + start.up,\cos t_{i,t}]U_i^t = F(P_i^t, U_i^t) \qquad (5)$$

We can form the Lagrange function as following:

$$\text{L(P,U, λ)=}\ F(P_i^t, U_i^t) + \sum_{t=1}^{T} \lambda^t (P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) \qquad (6)$$

The Lagrange relaxation procedure solves the unit commitment problem by "relaxing" or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through the dual optimization procedure as that attempts to reach the constrained optimum by maximizing the Lagrangian with respect to the Lagrange multipliers while minimizing with respect to the other variables in the problem; that is [5]:

$$q^*(\lambda) = \max_{\lambda^t} q(\lambda) \qquad (7)$$

Where

$$q(\lambda) = \min_{P_i^t, U_i^t}\ \text{L (P, U, λ)} \qquad (8)$$

This is done in two basic steps [5]:

Step 1 Find a value for each λ^t which moves *q(λ)* toward a larger value.

Step 2 Assuming that the λ^t found in step 1 are now fixed, find the minimum of $\mathcal{L}$ by adjusting the values of P^t and U^t

$$\mathcal{L} = \sum_{t=1}^{T}\sum_{i=1}^{N}[F_i(P_i^t) + start\cdot up,\cos t_{i,t}]U_i^t + \sum_{t=1}^{T}\lambda^t P_{load}^t - \sum_{t=1}^{T}\sum_{i=1}^{N}\lambda^t P_i^t U_i^t \qquad (9)$$

The second term above is constant and can be dropped (since the I' are fixed). Finally, we write the Lagrange function as:

$$\mathcal{L} = \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\{[F_i(P_i^t) + start.up, \cos t_{i,t}]U_i^t - \lambda^t P_i^t U_i^t\}\right) \qquad (10)$$

Here, we have achieved our goal of separating the units from one another. The term inside the outer brackets. Therefore, the problem can be solved separately for each generating unit, without regard to what is happening on the other generating units. The minimum of the Lagrangian is found by solving for the minimum for each generating unit over all time periods; that is [5]:

$$\min q(\lambda) = \sum_{i=1}^{N} \min \sum_{t=1}^{T}\{[F_i(P_i^t) + start.up, \cos t_{i,t}]U_i^t - \lambda^t P_i^t U_i^t\} \qquad (11)$$

subject to

$$U_i^t P_i^{\min} \le P_i^t \le U_i^t P_i^{\max} \quad \text{for t = 1,…….,T} \qquad (12)$$

- **Adjusting λ :**
  First of all, we should obtain an initial condition for the dual vector λ by using a few iterations of a dynamic programming algorithm. We consider the Lagrange multiplier to be fixed in each time period in order to ease the problem. Since λ is vector, each of which must be adjusted and treated separately [5].

$$\lambda^t = \lambda^t + [\frac{d}{d\lambda} q(\lambda)]\alpha \qquad (13)$$

We continue in these iterations until we get very close to the solution. We determine that using the "relative duality gap" $\frac{J* - q*}{q*}$ as a measure of closeness.

The flowchart shown below shows the procedure of the Lagrange relaxation method for solving the unit commitment problem:

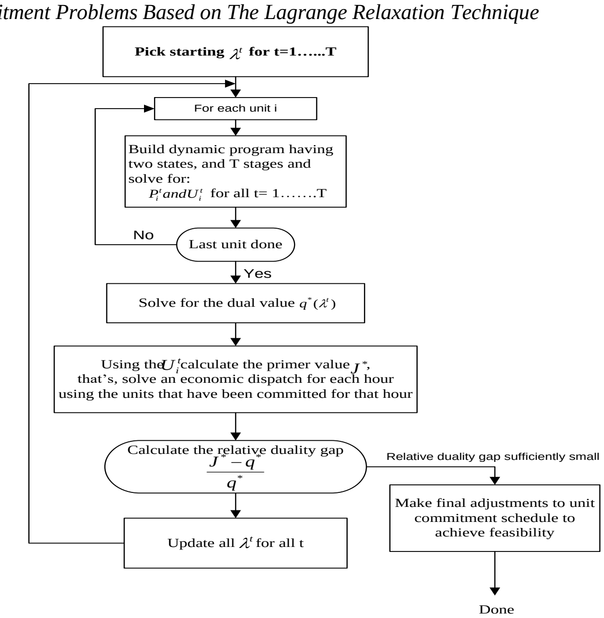

Figure 2. Basic Configuration of the LR Algorithm [5]

A great advantage of this method is that global system constraints such as demand-and-supply balance are relaxed by using Lagrange multipliers, and the problem is divided into subproblems for each generator. However, approximate solutions obtained by this method do not necessarily satisfy every constraint. In addition, when transmission losses are incorporated into demand-and-supply balance constraints, direct segmentation into sub-problems for each generator is no longer possible, which necessitates additional measures [7].

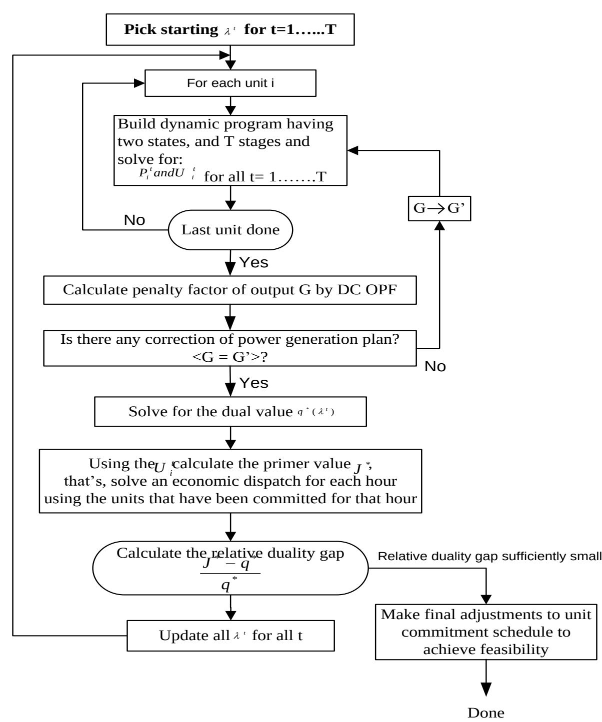

Figure 3. Basic Configuration of the LR Algorithm Taking into Account the Transmission Losses [7]

VI. NUMEICAL EXAMPLE

Units data including the generation cost (a, b and c) $F(P_i) = a + bP_i + cP_i^2$, the units' limits, minimum up and down time, initial states and the startup of each unit are presented in Table 1. Table 2 presents the load rate of each unit.

The unit commitment patterns obtained by Lagrange relaxation with and without regard to transmission losses are presented in Tables 3 and 4, respectively. In the tables, the columns represent time, and the rows the generator number [7].

The numbers (1 and 0) in Table 3 and 4 represent on/off states of units at different hours. Hour 0 represents the initial condition [8].

Table 1. Generator Model [7]

Unit No.	Unit characteristic constants			Output(MW)		MUT	MDT	Initial	Start up
	a (10^3 yen/h)	b (10^3 yen/MWh)	c (10^3 yen/MW²h)	P^{min}	P^{max}	(h)	(h)	state	cost
1	24.389	25.547	0.0253	2.4	12	1	1	-1	0
2	118.91	37.964	0.0156	4	20	1	1	-1	30
3	118.46	37.777	0.0136	4	20	1	1	-1	30
4	118.91	37.964	0.0116	4	20	1	1	-1	30
5	119.46	38.777	0.0106	4	20	1	1	-1	30
6	117.76	37.551	0.012	4	20	1	1	-1	30
7	118.11	37.664	0.0126	4	20	1	1	-1	30
8	81.826	13.507	0.0096	15.2	76	3	2	3	80
9	81.136	13.327	0.0088	15.2	76	3	2	3	80
10	81.298	13.354	0.009	15.2	76	3	2	3	80
11	81.626	13.407	0.0093	15.2	76	3	2	3	80
12	217.9	18	0.0062	25	100	4	2	5	100
13	219.78	18.6	0.006	25	100	4	2	5	100
14	218.34	18.1	0.0061	25	100	4	2	5	100
15	216.78	18.28	0.0059	25	100	4	2	-3	100
16	218.78	18.2	0.006	25	100	4	2	-3	100
17	216.78	17.28	0.0058	25	100	4	2	-3	100
18	218.78	19.2	0.007	25	100	4	2	-3	100
19	143.03	10.715	0.0047	54.25	155	5	3	5	200
20	143.32	10.737	0.0048	54.25	155	5	3	5	200
21	143.6	10.758	0.0049	54.25	155	5	3	5	200
22	259.13	23	0.0026	68.95	197	5	4	-4	300
23	259.65	23.1	0.0026	68.95	197	5	4	-4	300
24	260.18	23.2	0.0026	68.95	197	5	4	-4	300
25	260.58	23.4	0.0026	68.95	197	5	4	-4	300
26	261.18	23.5	0.0027	68.95	197	5	4	-4	300
27	260.08	23.04	0.0026	68.95	197	5	4	-4	300
28	176.06	10.842	0.0015	140	400	8	5	10	500
29	177.06	10.862	0.0015	140	400	8	5	10	500
30	176.06	10.662	0.0014	140	400	8	5	10	500
31	177.96	10.962	0.0016	140	400	8	5	10	500
32	310	7.4921	0.0019	100	450	8	5	10	800
33	311.91	7.5031	0.002	100	450	8	5	10	800
34	312.91	7.5121	0.002	100	450	8	5	10	800
35	314.9	7.5321	0.002	100	450	8	5	10	800

Table 2. Load Rate of Each Bus [7]

bus	rate	bus	rate	bus	rate	bus	rate	bus	rate
1	0.01	23	0.002	47	0.01	74	0.018	96	0.007
2	0.009	27	0.009	48	0.009	75	0.016	97	0.004
3	0.01	28	0.004	49	0.02	76	0.015	98	0.005
4	0.01	29	0.009	50	0.008	77	0.017	100	0.004
6	0.01	31	0.008	51	0.009	78	0.026	101	0.005
7	0.009	32	0.01	52	0.009	79	0.027	102	0.005
11	0.015	33	0.006	53	0.01	80	0.001	103	0.007
12	0.008	34	0.007	54	0.02	82	0.012	104	0.008
13	0.009	35	0.008	55	0.026	83	0.013	105	0.009
14	0.008	36	0.005	56	0.023	84	0.017	106	0.007
15	0.007	39	0.01	57	0.019	85	0.026	107	0.01
16	0.005	40	0.01	58	0.018	86	0.019	108	0.002
17	0.002	41	0.011	59	0.02	88	0.018	109	0.002
18	0.008	42	0.011	60	0.035	90	0.029	110	0.01
19	0.007	43	0.004	62	0.029	92	0.004	112	0.007
20	0.007	44	0.004	66	0.021	93	0.004	114	0.002
21	0.004	45	0.012	67	0.019	94	0.003	115	0.006
22	0.004	46	0.01	70	0.015	95	0.003	117	0.01
								118	0.02

Table 3. Unit Commitment Pattern (Transmission Losses Disregarded) [7]

Unit No.	Time (h) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
13	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
14	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
17	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
19	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 4. Unit Commitment Pattern (Transmission Losses Included) [7]

Unit	Time (h)																						
No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
12	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
13	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
17	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0
19	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

For the solution shown in Table 3, the number of Lagrange multiplier updates, the duality gap, and the overall operating cost were, respectively, 453, 0.8%, and 1219.025 million yen. In Table 4, the number of Lagrange multiplier updates, the duality gap, and the overall operating cost were, respectively, 468, 0.4%, and 1222.440 million yen. In addition, when the UC pattern obtained without regard to transmission losses was then corrected by using optimal power flow calculation, a supply deficiency occurred at time period 11, as marked in black. In other words, conventional Lagrange relaxation does not assure due allowance for transmission losses, hence unit commitment cannot be scheduled optimally. With this time period excluded, the overall operating cost amounted to 1178.579 million yen.

For the proposed method presented in Table 4, the number of Lagrange multiplier updates, the duality gap, and the overall operating cost were, respectively, 468, 0.4%, and 1222.440 million yen [7].

VII. CONCLUSION

This paper gives an introduction to the unit commitment problem. Most of the important constrain that can affect the solution of the unit commitment problem are stated and discussed. Also, the paper introduces two methods through which the UC problem could be solved (Dynamic Programming and Lagrange Relaxation), and some advantages and disadvantages of each method are discussed. The problem formulation of both methods is presented in this paper. A comprehensive example is given to solve the unit commitment problem using Lagrange relaxation. Also, an improved method of LR is used to account for the transmission losses.

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