



THE EFFECT OF SUCTION ON THE BOUNDARY LAYER SEPARATION OF A RETARDED BOUNDARY LAYER FLOW

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Index Terms

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Abstract

Boundary layer separation is one of the most drawback phenomena that reduces or even eliminates the lifting force generated by the flow over an airfoil. Boundary layer flow separation occurs due to an increasing adverse pressure gradient in the flow direction. Boundary layer theory fails to predict and describe this singularity where shear stress vanishes in the vicinity of the wall. To generate this phenomenon for a laminar flow over a flat plate in the presence of a pressure gradient, the linearly retarded flow of Howarth has been used. The decelerating non-similar free-stream velocity profile becomes increasingly S-shaped and finally separates downstream. The numerical solution of the considered flow reproduced the theoretical result reported by Howarth for the location of the separation point which is approximately ($x_{sep}/L_x \approx 0.125$). A suction strategy is applied over the wall which tends to shift the location of the separation point far downstream or completely suppresses its formation.

تأثير عملية المص على انفصال الطبقة الجدارية لسريان الطبقة الجدارية المبطل

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الكلمات المفتاحية

سريان الطبقة الجدارية، انفصال الطبقة الجدارية، ديناميكا الموائع الحسابية، التحكم في ظواهر التدفق

الملخص

يعد انفصال الطبقة الجدارية أحد أكثر الظواهر السلبية التي تقلل أو حتى تلغي قوة الرفع التي ينتجها التدفق على الجناح. يحدث انفصال تدفق الطبقة الجدارية بسبب زيادة تدرج الضغط المعاكس في اتجاه التدفق. تفشل نظرية الطبقة الجدارية في التنبؤ ووصف هذا التفرد حيث يتلاشى إجهاد القص في المنطقة الملاصقة للجدار. لتوليد هذه الظاهرة للتدفق الرقائقي على سطح لوح و في وجود تدرج للضغط، تم استخدام التدفق المبطل خطياً لهوارث. يأخذ نمط حقل السرعة للتدفق المبطل شكل حرف S بشكل متزايد في اتجاه السريان ثم ينفصل أخيراً. من خلال إجراء المحاكاة الحسابية لهذا التدفق تم الحصول على نفس القيمة لموضع نقطة انفصال الطبقة الجدارية والتي هي تقريباً $x_{sep}/L_x \approx 0.125$. تم بعد ذلك تطبيق إستراتيجية الشفط أو المص على الجدار والتي نتج عنها تأخير حدوث انفصال الطبقة الجدارية وإبعاد موضع نقطة الانفصال عن موضعها في كل حالة أو قمع تكونها نهائياً.

I. INTRODUCTION

One of the major instability problems in boundary layer flows is the tendency of the boundary layer to separate from the surface of the body over which it flows. The phenomenon is known as boundary layer separation and is affected directly by the body shape, the skin friction exerted by the fluid on the wall and any condition that could lead to the formation of an adverse pressure gradient downstream of the flow. Flow separation has been noticed since the discovery of the boundary layer by Prandtl 1950 but has not been exclusively studied and understood till the mid of the 20th century. Many research works have discussed the physics behind the boundary layer separation and their effects on the flow behavior leading to the conclusion that the pressure gradient along the wall acted together with the friction along the wall to govern the separation process [1]. Since the pressure gradient inside the boundary layer is determined by that of the free stream region, the existence of an adverse pressure gradient in the outer flow will propagate to the near wall region. The presence of an adverse pressure gradient downstream of the flow overcomes the momentum contained within the flow and the separation of the boundary layer occurs. At the location where boundary layer separation occurs, both the pressure gradient and the curvature of the velocity profile have a positive sign. Since the curvature of the velocity profile will maintain a negative sign at the edge of the boundary layer, the velocity profile will go through an inflection point somewhere between the wall and the boundary layer edge. A positive velocity profile curvature makes fluid layers adjacent to the body surface to take a reversed direction opposite to the main flow stream. When separation takes place the velocity gradient near the body surface become negative and consequently the shear stress profile changes its sign and direction. This behavior is traditionally used to define the boundary layer separation point, the point where the velocity gradient vanishes [2]. In other words the separation point is defined as the position on the wall where shear stress comes in vertically to zero. (i.e $\partial \tau / \partial n = 0$) [3].

This is known as Goldstein singularity, after [4] who showed that in the boundary layer theory the wall shear stress has a square root profile that tends to zero near separation, and no real solution can then be derived downstream of separation which represents a fundamental limitation of the boundary layer theory. Theoretical series solution attempts of the boundary layer equations for non similar free stream distributions revealed also a fundamental limitation of the boundary layer theory at and beyond the singularity, where the normal velocity component (v) becomes infinite at this location, thereby

violating the assumption that the B.L is thin [2]. Theoretical determination of the position of the boundary layer separation by solving boundary layer equations can only be carried out using a priori known experimental or theoretical pressure gradient that can produce unfavorable pressure gradient far downstream. Evidence also suggested that the location of the boundary layer separation is distinct for each kind of flow [4]. For laminar boundary layer flow over a flat plate as has been estimated theoretically by [5], the boundary layer separation occurs approximately at ($x_{sep}/L_x \approx 0.125$) [1].

In many aerodynamic applications where boundary layer flow arises, flow separation can often result in an increased drag, particularly pressure drag which is caused by the pressure differential between the front and rear surfaces of the object as it travels through the flow. Flow separation can also speed up the transition process of the flow to the turbulent regime and leads to the formation of the vortex shedding streets which can produce serious vibration problems to the object. For this reason much effort and research was devoted to develop control techniques that can delay or even suppress flow separation process. Many techniques have been developed during the last two decades which can be classified into two major categories, the passive and active flow control approaches [6]. In the passive flow control approach the control acts upon the flow through surface or geometric modifications, which can delay flow separation and keep the local flow attached for as long as possible. For active flow control approach, the flow is subjected to a suitable external actuation that can modify the flow and delay or prevent flow separation. One of the widely used active flow control techniques is the concept of suction and blowing of some amount of fluid through the surface of the body over which the flow occurs. Different types of active flow control strategies, such as optimal control strategy, heuristic control strategy, adaptive control strategy, AI based active control strategies [7], [8], [9] have been applied to control a wide spectrum of flow problems. Depending on the active flow control strategy used, the characteristics of blowing and suction through the body surface can be determined in order to bring the controlled flow to the desired state.

In the current study the retarded laminar boundary layer flow problem over a slender flat plate has been considered to generate flow separation far downstream. The retarded flow has been established by imposing a simple decelerating non similar free-stream velocity profile [5]. The decelerating free stream flow induces an adverse pressure gradient far downstream where flow separation is likely to occur. The problem has been treated numerically by solving the boundary layer equation using a simple decelerating non-similar free-stream velocity profile. The location of the boundary layer separation point has been investigated and compared to its

corresponding theoretical values for the same type of flow. The flow is then subjected to a distributed suction over the surface of the plate to study its effect on the boundary layer separation process.

II. PROBLEM STATEMENT AND FLOW CONFIGURATION

A numerical simulation of a two dimensional incompressible boundary layer flow over a flat plate was considered in order to simulate and study the characteristics of the boundary layer separation process. The flow is governed by the boundary layer equations given by:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

Since the free stream velocity field $U = U(x)$ outside the boundary layer is related to the pressure field $p(x)$ by Bernoulli's theorem for incompressible flows, the pressure field can be described as:

$$\frac{dp}{dx} = -\rho U \frac{dU}{dx} \quad (3)$$

The governing equations modify to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

Subjected to the B.C's

$$u(x,0) = v(x,0) = 0 \quad \text{and} \quad u(x,\infty) = U(x) \quad (6)$$

The flat plate is of a length L_x and an infinite width with a negligible thickness, that lies in the $x-y$ plane, and whose two edges correspond to $x = 0$ and L_x "Fig. 1.". The plate is supposed to be subjected to a retarded laminar flow characterized by a linearly decelerated velocity profile given by:

$$U(x) = U_0 \left(1 - \frac{x}{L_x} \right) \quad (7)$$

In the first case, where no blowing or suction was applied, the flow is subjected to the no slip boundary conditions at the wall ($u = v = 0$).

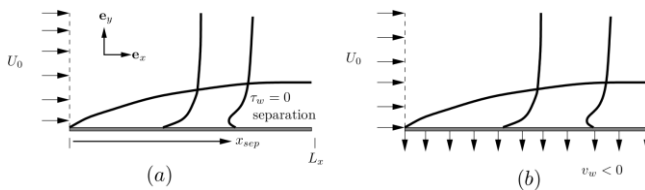


Fig. 1: Schematic layout of the retarded boundary layer flow without (a) and with (b) uniform suction at the wall.

Boundary layer flow is characterized by the presence of the shear layer in the boundary layer region, which when accompanied by an adverse pressure gradient downstream of the flow will make the fluid layers close to the wall to reverse their flow direction leading to the separation of the B.L. In order to create an adverse pressure gradient downstream of the flow for the B.L simulations, the retarded flow velocity profile "Eq. (7)" has been imposed to the free stream region which makes the flow decelerates in the flow direction. This will create an adverse pressure gradient downstream which propagates into the near wall region making fluid layers close to the wall lose much of their kinetic energy and start moving in a reversed direction to the main flow stream (separation). Since the boundary layer flow simulation diverges as the implicit marching technique reaches the separation zone, where the flow admits singular behavior, a procedure of suction ($v_w < 0$) on the wall has been applied in order to push the separation point further downstream.

As the separation of the B.L occurs at a given downstream location, the simulation procedure has to be stopped using a suitable criterion exactly before separation takes place. The value of the wall shear stress as it becomes zero or negative can be considered the best choice that can indicate the position where separation is likely to occur. It has been adopted as a stopping criterion for the simulations reported here. This will allow the exact determination of the location of the separation point and to inspect the effect of blowing and suction (local and/or distributed) procedure on the boundary layer separation.

III. NUMERICAL PROCEDURE

The governing equations "Eq. (4)", "Eq. (5)" have been discretized using uniform grid in the stream-wise direction and a non-uniform grid with a hyperbolic tangent profile in the normal direction to the wall e_y [10]. The non-uniform grid profile in e_y direction clusters grid points in the flow regions near the wall where the flow field undergoes big variations and changes. Grid resolution has been kept the same throughout all simulations realized in this work. The resolution procedure starts with the discretization of the governing equations where derivatives in the e_y direction have been discretized using the central finite difference scheme which is second order accurate in y . Other terms have been discretized using the forward finite difference scheme which is first order accurate. The solution procedure admitted the use of an implicit marching technique following the streamwise direction e_x . Implicit marching has been adopted to solve the boundary layer equations and model the flow since it is unconditionally stable and imposes no restrictions to the choice of the marching step size [11]. The discretized governing equation on the non-uniform grid using an implicit marching scheme along e_x direction is as follows:

$$u_{i,j} \frac{u_{i+1,j} - u_{i,j}}{\Delta x} + v_{i,j} \frac{u_{i,j+1} - u_{i,j-1}}{(\Delta y_{j+1} + \Delta y_j)} = \frac{U_{i+1}^2 - U_i^2}{2\Delta x} + v \left[\frac{\frac{u_{i+1,j+1} - u_{i+1,j-1}}{\Delta y_{j+1}} - \frac{u_{i+1,j} - u_{i+1,j-1}}{\Delta y_j}}{\frac{\Delta y_{j+1} + \Delta y_j}{2}} \right] \quad (8)$$

This can be converted into a discrete tri-diagonal form:

$$\begin{aligned} & \frac{-2\nu \Delta x}{\tilde{\Delta y} \cdot \Delta y_j} u_{i+1,j+1} + \frac{2\nu \Delta x}{\tilde{\Delta y}} \left(\frac{1}{\Delta y_{j+1}} + \frac{1}{\Delta y_j} \right) u_{i+1,j} - \\ & \frac{2\nu \Delta x}{\tilde{\Delta y} \cdot \Delta y_j} u_{i+1,j-1} = u_{i,j} \cdot u_{i,j} - \frac{\Delta x}{\tilde{\Delta y}} v_{i,j} (u_{i,j+1} - u_{i,j-1}) \quad (9) \\ & + \frac{1}{2} (U_{i+1}^2 - U_i^2) \end{aligned}$$

Where $\tilde{\Delta y} = \Delta y_{j+1} + \Delta y_j$

In a compact form the discretized equation corresponds to the following tri-diagonal matrix system:

$$Bu_{j+1} + Au_j + Cu_{j-1} = RHS_j \quad (10)$$

The implicitly discretized momentum equation at grid points along the normal direction performs a tri-diagonal matrix system for each downstream location. The constructed tri-diagonal matrix system can be solved by any iterative means or using matrix inversion to get the velocity component $u(x_i, y_j)$ at every mesh point.

There are no instability constraints regarding the value of the marching step Δx . The resulted streamwise velocity component can then be used to predict the values of the normal velocity component $v(x_i, y_j)$ at each grid site using the discretized continuity equation.

$$\frac{u_{i,j} - u_{i-1,j}}{\Delta x} - \frac{v_{i,j} - v_{i,j-1}}{\Delta y} \approx 0 \quad (11)$$

Since eq. "Eq. (11)" evaluates the two terms at different grid sites, the first term is evaluated at the level j and the second term at $j - \frac{1}{2}$, the former discretization gives poor numerical accuracy. To circumvent this difficulty a method suggested by [12] is used. In his approach $\frac{\partial u}{\partial x}$ is moved to level $j - \frac{1}{2}$ by using the average value:

$$\frac{\partial u}{\partial x} \approx \frac{1}{2} \left(\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{u_{i,j-1} - u_{i-1,j-1}}{\Delta x} \right)$$

This expression is then used to solve for the next vertical velocity component:

$$v_{i,j} = v_{i,j-1} - \frac{\Delta y}{2\Delta x} (u_{i,j} - u_{i-1,j} + u_{i,j-1} - u_{i-1,j-1}) \quad (12)$$

The resolution starts by defining flow parameters and generating the grid. For the case when no plowing or suction is applied at the wall, the no slip condition is imposed at the wall. Since the flow decelerates in the downstream direction, an adverse pressure gradient develops downstream and boundary layer separation takes place somewhere before the end of the plate. The solution of the boundary layer equations presents a singularity at the separation point which will make the solution diverge. Therefore, the simulations will stop as the local shear stress vanishes or change in sign which indicates that the location of the separation point was achieved. In order to study the effect of suction procedures on the boundary layer separation, the suction is applied using a set of different negative values for the normal velocity component at the wall [13].

Since the implicit scheme used is based on using the streamwise direction as an implicate marching direction, the discretized boundary layer equations at all the nodes in the normal direction perform a tri-diagonal system and that is for each downstream station. These matrix systems are solved consequently starting from the leading edge of the plate to the trailing edge using Thomas algorithm known as TDMA (Tri-diagonal matrix algorithm) [14]. The predicted values of streamwise velocity component at each downstream location are used to update the normal velocity component using the discretized continuity equation.

IV. SIMULATION PARAMETERS

Flow properties and parameters used in the simulations are described by the "Table 1,". The dynamic viscosity could be determined according to the suggested values of Reynolds number ($Re = \frac{\rho U_0 L_x}{\mu}$) which is defined based

on the inlet velocity U_0 and the plate length L_x . The parameters N_x, N_y denote the number of grid points following e_x and e_y directions respectively. Boundary layer flow simulations have been conducted using the same value of inlet velocity ($U_0 = 1 \text{ m/s}$). The effect of Reynolds number on the position of the separation point has been verified by running laminar flow simulations with the no slip boundary condition at the wall for 10 different values of Reynolds number (see "Table 1,"). The same numerical experiments have then been applied using a distributed suction procedure over the entire wall. The suction velocity has been evaluated based on the fact that the reduced suction velocity v_w/U_0 should be of an

order of magnitude $O(\sqrt{1/Re})$ [1]. A parametric study has been also conducted in order to determine the best value of the applied normal velocity distribution at the wall which suppresses the separation and minimize the overall drag coefficient.

V. RESULTS & DISCUSSIONS

Firstly, the retarded boundary layer flow simulations for some selected values of Reynolds number were conducted using the no slip B.C at the wall. The considered flows are suggested to develop flow separation far downstream on the wall. When there is no suction, the location of the separation point along the streamwise direction does not change and exhibits the same location for all Reynolds number. All flow simulations without suction have shown that the flow separation occurs exactly at $(x_{sep}/L_x \approx 0.125)$, which is in excellent agreement with the theoretical value reported by [1]. “Figure 2,” Shows the location of the separation point for the boundary layer flow simulations at the considered range of Reynolds number and suction intensity. Clearly, the location of the separation point is displaced forward downstream, and the displacement magnitude increases with Reynolds number and with suction intensity. The larger the Reynolds number and suction intensity the far downstream the separation point has been displaced. Flow characteristics up to the location of the separation point have been plotted against the Blasius-type function $(\eta = y\sqrt{U_0/\nu x})$, and investigated in order to compare them with their corresponding values as the suction procedure is applied at the wall. Among the investigated flow characteristics are the streamwise velocity component, the normal velocity component, the shear stress at the wall, the drag coefficient and the location of the separation point within the flow. To resume about the effect of suction on boundary layer separation process the results are analyzed in details for either the two cases with and without suction at the wall.

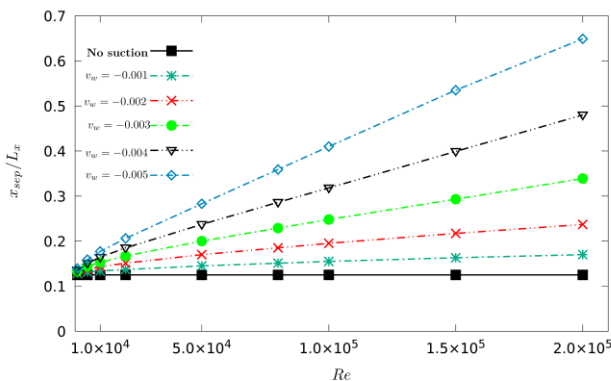


Fig. 2: Separation point location at different Reynolds number and at different values of suction velocity.

A. Velocity Profiles

The retarded B.L flow with no-slip B.C at the wall shows a stream-wise velocity profile with a decreasing gradient in the normal direction which vanishes in the near wall region as boundary layer separation takes place. In this case where there is no suction at the wall, the velocity profile remains similar and flow characteristics do not change with Reynolds number as indicated by “Fig. 2,”. For the case where no suction is applied, “Fig. 3,” shows

the stream-wise velocity profile of the retarded flow at $(Re = 2 \times 10^4)$ for different distances from the leading edge up to the location of the separation point. The velocity profile exhibits a progressively decreasing slope closer to the separation point, and tends to take gradually an S-shaped appearance. Once suction is applied, as demonstrated by “Fig. 4,” which shows streamwise velocity profiles at different locations along the wall for suction velocity $(v_w = -0.001)$. The velocity profile became significantly influenced by the suction procedure, especially in the vicinity of the separation point. As suction intensity increases, the gradient of the stream-wise velocity profile decreases slowly in the near wall region “Fig. 5,” which delays the separation event and the stronger the suction, the farther away the separation event occurs. Velocity profiles are no more similar when the suction procedure is applied; they lose their parabolic shape with a decreasing velocity gradient close to the separation point which has been pushed further downstream as strong as the suction intensity is increased.

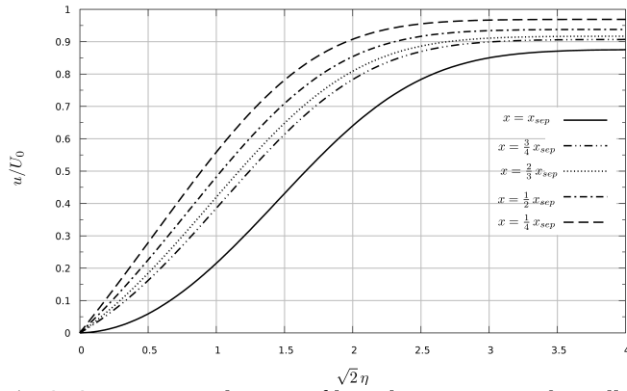
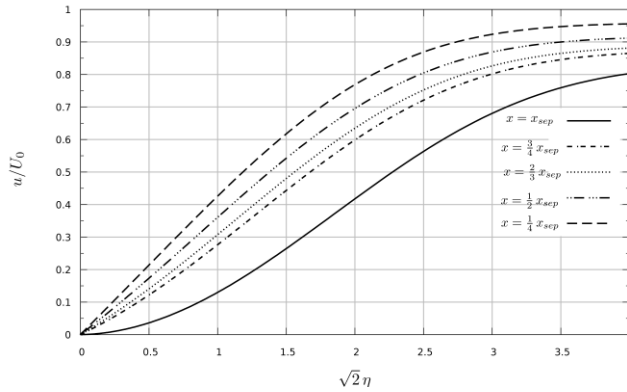
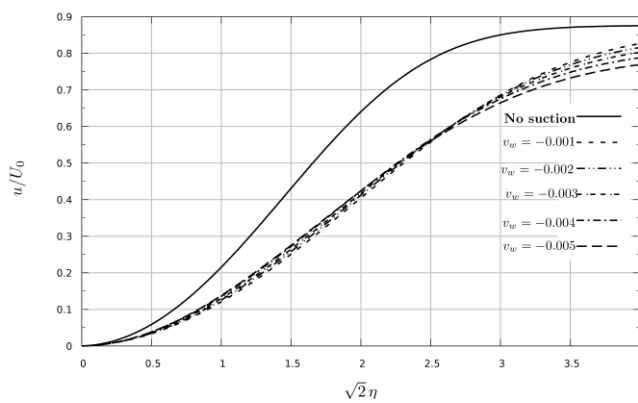
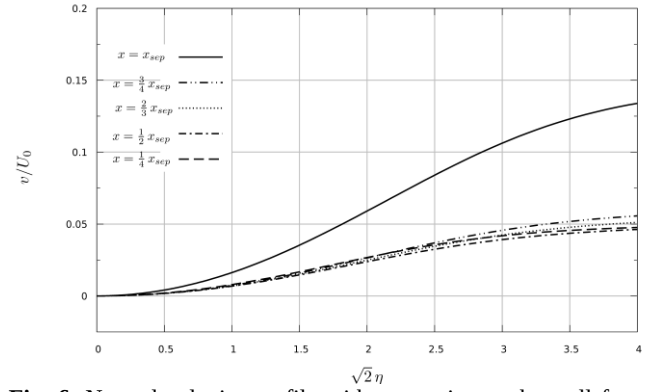
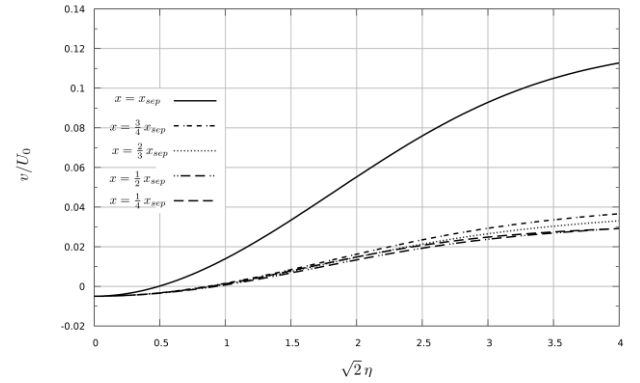
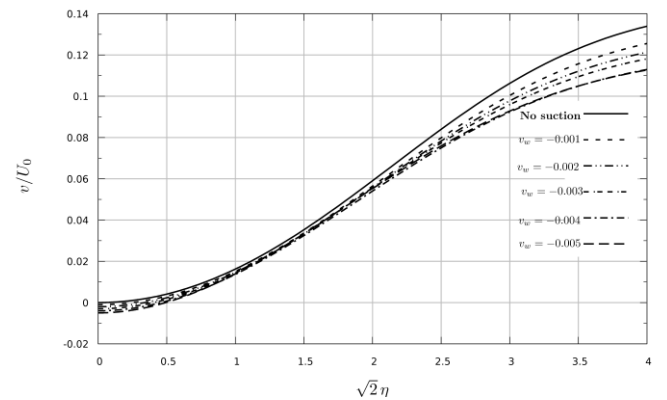
For the case of the retarded BL flow without suction “Fig. 6,” the normal velocity component increases in the normal direction as the flow moves towards the separation point. This behavior which is clearly described by “Fig. 7,” where normal velocity component at different locations at the wall are monitored for suction velocity $(v_w = -0.005)$, indicating that, the normal velocity component is zero in the tiny region close to the wall and takes increasingly large values in the vicinity of the separation point. A kind of blowing mechanism arises in the interior of the boundary layer, as described by triple deck theory [15], which makes the boundary layer thicker and creates a deflection of the stream lines in the main flow and in turn induces a pressure disturbance that transmitted back in the boundary layer regions close to the wall promoting velocity disturbance and hence flow separation. Therefore the hypothesis upon which resides the boundary layer theory collapse and become no more valid in describing the physics of the flow as the singularity occurs. By increasing suction intensity, normal velocity component takes lower magnitudes than the case without suction “Fig. 8,” thereby interrupting the mechanism that induces the adverse pressure gradient and hence delaying the occurrence of the separation event.

A. Shear stresses

“Fig. 9,” illustrates the local shear stress resulted from a retarded BL flow simulations at $Re = 2 \times 10^4$ using different values of the suction velocity. Clearly the local shear stress has a decaying profile and vanishes exactly at the separation point. By increasing the suction strength, the point where the value of the shear stress vanishes is displaced further downstream. The suction process acts at the wall with a mechanism that tries to keep the flow attached to the wall which slightly increases the local shear stress. Consequently, the zero local wall shear stress has been displaced further downstream.

TABLE 1: SIMULATION PARAMETERS

Re	U_0 (m/s)	ρ (kg/m ³)	L_x (m)	L_y (m)	$\Delta x \times 10^3$	$\Delta y_{\min} \times 10^3$	$\Delta y_{\max} \times 10^3$	N_x	N_y
$10^2 : 2 \times 10^5$	1	1	6	2	6	4.2	6.4	1000	500

**Fig. 3:** Stream-wise velocity profile without suction at the wall for $Re = 2 \times 10^4$ and different locations at the wall up to separation point.**Fig. 4:** Stream-wise velocity profile with suction ($v_w = -0.001$) for $Re = 2 \times 10^4$ and different locations at the wall up to separation point.**Fig. 5:** Stream-wise velocity profile with different suction velocities for $Re = 2 \times 10^4$ and at location extremely close to the separation point.**Fig. 6:** Normal velocity profile without suction at the wall for $Re = 2 \times 10^4$ and different locations at the wall up to separation point.**Fig. 7:** Normal velocity profile with suction ($v_w = -0.005$) for $Re = 2 \times 10^4$ and different locations at the wall up to separation point.**Fig. 8:** Normal velocity profile of a retarded BL flow with different suction velocities for $Re = 2 \times 10^4$ and at location extremely close to the separation point.

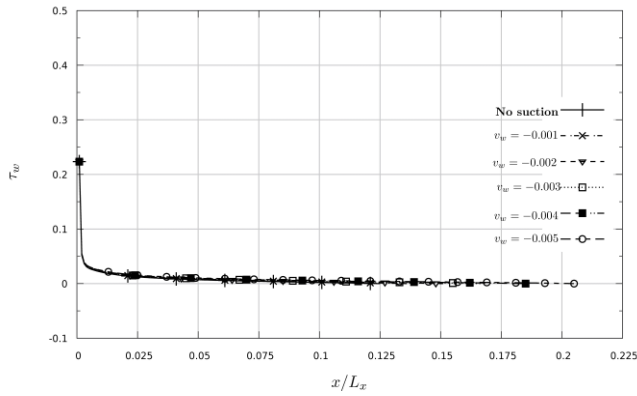


Fig. 9: Local shear stress at $Re = 2 \times 10^4$ with different suction velocities

“Fig.10,” shows the drag coefficient ($C_D = \bar{\tau} / \frac{1}{2} \rho U_0^2 A$) with $\bar{\tau}$ represents the mean wall shear stress at the wall, for a retarded BL flow simulations at different suction velocity and different Reynolds numbers. The results show a slight decrease in the drag coefficient by increasing suction intensity compared to the case without suction, but the effect of increasing Reynolds number has a big influence on decreasing the amount of the total drag over the wall surface area up to the separation point.

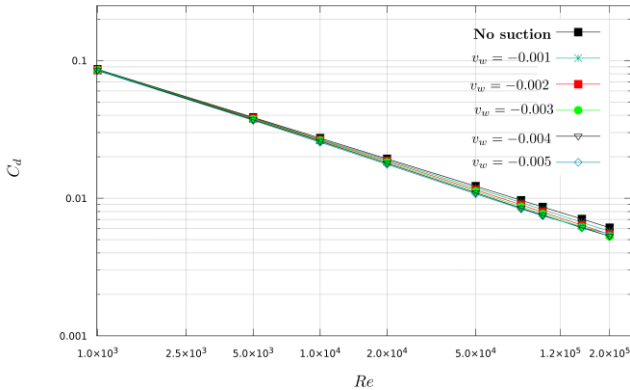


Fig. 10: Drag coefficient of a retarded BL flow with different suction velocities for $Re = 2 \times 10^4$ over a distance goes up to the separation point.

VI. CONCLUSION

Boundary layer separation phenomenon was generated using a retarded boundary layer flow over a flat plate. The decelerated flow induces an adverse pressure gradient in the flow direction which slows down the streamwise velocity field in the inner region of the boundary layer and provokes the singularity. Different retarded flow simulations at 10 laminar values of Reynolds number were investigated. These simulations indicated that the separation point location is not affected by the Reynolds number and stayed the same at ($x_{sep}/L_x \approx 0.125$). Flow simulations revealed a sort of blowing mechanism in the main region of the boundary layer where normal velocity component acquires large values which contributes to the formation of normal

pressure gradient that disturbs the stream-wise velocity component in the inner region of the BL. In order to examine the effect of a distributed suction at the wall on the flow dynamics that associated to the formation of boundary layer separation, the simulations were repeated using non zero normal velocity boundary condition at the wall. The results indicate a strong effect of the suction procedure that led to delaying the location of the singularity far down-stream as long as the Reynolds number was increased. The results showed also that by increasing Reynolds number the wall shear stress obeys an asymptotic decay up to the separation point where it vanishes. Simulations results concluded also that the suction intensity contributes strongly to the delaying mechanism of the BL separation.

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