Perturbation-based Extremum Seeking Control Design for a Class of Nonlinear Systems

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Abstract — **In this paper, we try to address the problem of output (performance) function by applying the Perturbation-based Extremum Seeking Control (PESC) approach to the observer of the Single-Input Single-Output (SISO) nonlinear systems. In this work, PESC is applied so that the performance function can reach its maximum value. We apply tow controllers design that will take care of maximizing of the cost function. First controller is designed in the availability of full and unknown variables which are fed to the objective function, and the second controller is designed when the full state availability is removed, and these variables become known by applying the High Gain Observer (HGO) model to estimate the system variables. The construction of a seeking algorithm is used to drive the system variables and the observer output to the desired set-points that maximize the value of an objective (performance) function. In addition, Lyapunov's stability theorem and the perturbation theory including the averaging method are used in the design of the extremum seeking controller structure to check the stability of the system. Finally, the simulation results show the performance of the proposed procedures.**

Index Terms: **Nonlinear system, high gain observer, and extremum seeking control.**

I. INTRODUCTION

All of the issues in control systems need to be addressed in order to survive in today's market. There are many techniques to achieve or to work on the above tasks or goals. The technique is: providing an effective control technique. It is well known that traditionally control includes two main objectives: modeling of the process (plant) and design the control algorithm for the process (plant). In the literature, researchers have spent much effort for modeling systems such as; linear and nonlinear systems. Nonlinear control systems are used as a feedback to produce a control signal mathematically based on other variables.

The output from this control system into the controlled process may be in the form of a directly variable signal. Sometimes this is not feasible and so, after calculating the current required corrective signal, the control system may repeatedly switch an actuator, such as a pump, motor or heater, fully on and then fully off again, regulating the duty cycle using pulse-width modulation.

The study of extremum seeking control and its application can be traced back to the early 20th century. Many papers have reviewed earlier works in this field and shown good results [1-7]. Solar cells, blade orientation control in water turbines and wind mills, combustion processes in engines and generating plants are among the main applications.

The goal of extremum seeking is to find the operating setpoints that maximize or minimize an objective function. Since the early research work on extremum control in the 1920's (Leblanc 1922), many successful applications of extremum control approaches have been reported (e.g., (Vasu 1957), (Astrom and Wittenmark 1995), (Sternby 1980) and (Drkunov et al. 1995)). Recently, Krstic et. al ((Krstic 2000), (Krstic and Deng 1998)) presented several extremum control schemes and stability analysis for extremum-seeking of linear unknown systems and a class of general nonlinear systems ((Krstic 2000) and (Krstic and Deng 1998)) [8]. Krstic and Wang extended his study to an extremum seeking feedback scheme for general nonlinear systems with stability analysis, but not on the observer of the nonlinear system. Fu, Lina focuses on model-based extremum seeking control, and develops control design to integrate system control and optimization without time-scale separation. It aims to achieve faster convergence as well as better robustness for general nonlinear systems [9]. The inclusion of a dynamic compensator in the extremum seeking algorithm which improves the stability and performance properties of the method is presented by Miroslav Krstic in [10]. After this little literature review on PESC, we can conclude that the work of the PESC of the observer of the nonlinear systems under the required conditions is not done yet, so in this work we design the controller that can take care of the minimum of the cost

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function that can be applied on the observer of the nonlinear systems.

II. SISO NONLINEAR SYSTEMS

In this section, we work on the state feedback linearizable (SFL) systems as Raul Ordonez and Chunlei Zhang have done in chapter 5 in [11]. In other words, the difference here is that we work and design a different controller that lets the cost function reaches its maximum.

A. Proplem Statement

Consider the nonlinear system

$$\dot{x} = f(x) + g(x)u. \tag{1}$$

The output is measured by full states and considered as the cost function that we are going to maximize. This cost function is not actually the output of the system, and it is described as

$$y = J(x) \tag{2}$$

where $x \in \Re^n$ is the system states $u \in \Re$ is the input, and $y \in \Re^n$ is the performance output which must be convex on the domain D where $D \in \Re^n$ which means that the stationary point condition becomes a necessary and sufficient condition to identify a global maximizer.

Assumptions 2.1.1. The nonlinear system should be state feedback linearizable, and it should satisfy

- $\dfrac{dT_i(x)}{dx} g(x) = 0,$
- $\dfrac{dT_h(x)}{dx} g(x) \neq 0,$
- $T_j(x) \dfrac{dT_{(i-1)}(x)}{dx} f(x),$

where $i = 1, 2, \cdots, n-1,$ and $j = 2, 3, \cdots, n,$ and also

$$T(x) = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} T_1(x) \\ T_2(x) \\ \vdots \\ T_n(x) \end{bmatrix}.$$

It is possible here to devise a controller $u(t)$ and a coordinate transformation $z = T(x)$ such that the application of the controller $u(t)$ results in a linear system in z-coordinates. Actually, all this can happen depending on the actual system dynamics. As we mentioned in the *Assumptions 2.1.1*, the system in state feedback linearizable, so the system can be transferred to the form $z = T(x)$ which is called a diffeomorphism if the $T : \Re^n \to \Re^n$ is continuous and differentiable (C^1) and also $T^{-1}(x)$ exists.

The diffeomorphism here is going to result in a transferred system in the dynamics of z-coordinates to the form:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= \hat{f}(z) + \hat{g}(z)u \end{aligned} \tag{3}$$

Rewriting the system in another form will lead us to the form:

$$\dot{z} = Az + B(\hat{f}(z) + \hat{g}(z)u) \tag{4}$$

where $(A;B)$ is controllable canonical form which is

$$A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}, \qquad B = \begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix},$$

and $\hat{g}(z) = \hat{g}(T(x))$ is non-singular for all x. The system (3) is still a nonlinear system, and this form is called a chain of n integrators. So, we need to design a controller so that the system (3) can be linearizable.

B. Perturbation Based System Extremum Seeking control for SFL

To design the controller, we have to take care of some errors dynamics in the system. In this case of problem, we have to deal with some error dynamics, and these two errors should go as close as possible to zero when the time goes to infinity.

Suppose that we know a smooth control law

$$u(t) = \beta(z, \theta) = -\frac{1}{\hat{g}(z)}[\hat{f}(z) + Kz + \theta] \tag{5}$$

parameterized by a scalar parameter θ , and $K = [k_1 \ \cdots \ k_n]^T$. The assumption that θ, ϕ and y are scalars is also for simplicity; it can be trivially removed by using vectors of appropriate dimensions.

After we substitute the controller, the system becomes

$$\dot{z} = \bar{A}z + \bar{B}\theta, \tag{6}$$

where $\bar{A}$ and $\bar{B}$ are defined as

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

The analysis that follows employs the method of averaging. Let

$$\zeta = \frac{\omega_h}{s+\omega_h} J(z), \quad (7)$$

where $J(z) = y$ is the cost function that will be maximized, and $\frac{s}{s+\omega_h}$ is High-Pass Filter (HPF). Then, the signal after the washout filter (High-Pass Filter) can be expressed as

$$\frac{s}{s+\omega_h} y = y - \frac{\omega_h}{s+\omega_h} y = y - \zeta = \Delta. \quad (8)$$

The closed-loop system in (6) and

$$\dot{\theta} = \gamma(z, \hat{\phi}), \quad (9)$$

then have equilibria parameterized by θ and $\hat{\phi}$. We make the following assumptions about the closed-loop system.

Assumption 2.2.2. There exists a smooth function $l : \Re \to \Re^n$ such that

$$\begin{aligned} \bar{A}z + \bar{B}\theta = 0 \\ \gamma(z, \hat{\phi}) = 0 \end{aligned} \quad (10)$$

if and only if $z = \bar{l}(\theta)$ or $z = l(\hat{\phi})$.

Assumption 2.2.2. For each $\hat{\phi} \in \Re$ and $\theta \in \Re$, the equilibrium $z = l(\hat{\phi})$ and $z = \bar{l}(\theta)$ of the system (10). From the system (6) it is clear that all the equilibrium of z is zero except z_1, then we work on the equilibrium $z = l(\hat{\phi})$ which is locally exponentially stable with decay and overshoot constants uniform in $\hat{\phi}$.

Hence, we assume that we have a control law (5) which is robust with respect to its own parameter θ in the sense that it exponentially stabilizes any of the equilibria that θ may produce. Except for the requirement that *Assumption 2.2.2* holds for any $\theta \in \Re$ (which we impose only for notational convenience and can easily relax to an interval in $\Re$), this assumption is not restrictive. It simply means that we have a control law designed for local stabilization and this control law need not be based on modeling knowledge of either $(\bar{A}z + \bar{B}\theta)$ or $l(\hat{\phi})$.

The next assumption is central to the problem of peak seeking.

Assumption 2.2.3. There exists $\phi^* \in \Re$ such that

$$(Jol)'(\phi^*) = 0, \quad (11)$$

$$(Jol)''(\phi^*) < 0. \quad (12)$$

Thus, we assume that the output equilibrium map $y = J(l(\hat{\phi}))$ has a maximum at $\hat{\phi} = \phi^*$. Our objective is to develop a feedback mechanism which maximize the steady-state value of y but without requiring the knowledge of either ϕ^* or the functions J and l. Our assumption that is Jol has a maximum without loss of generality.

In the feedback scheme, it employs a slow periodic perturbation $a\cos(\omega t)$ which is added to the signal $\hat{\phi}$ that is estimated by ϕ^*, and a also needs to be small. The high-pass filter $s / s + \omega_h$ eliminates the "DC component" of y, so the product of the HPF will be extracted by Low-Pass Filter (LPF) which is $\omega_l / s + \omega_l$.

The selection of design parameters is indeed intricate, so these parameters are selected as

$$\omega_h = \omega\omega_H = \omega\delta\omega'_H = O(\omega\delta), \quad (13)$$

$$\omega_l = \omega\omega_L = \omega\delta\omega'_L = O(\omega\delta), \quad (14)$$

$$k = \omega K = \omega\delta K' = O(\omega\delta), \quad (15)$$

where ω and δ are small positive constants and ω'_H, ω'_L, and K' are $O(1)$ positive constants.

From (13) and (14) we see that the cut-off frequencies of the filters need to be lower than the frequency of the perturbation signal. In addition, the adaptation gain k needs to be small [12].

Now, we analyze the stability of the system. But before the analysis, we summarize the system as follows

$$\begin{aligned} &\dot{z} = \bar{A}z + \bar{B}\theta \\ &\dot{\theta} = \gamma(z, \phi + a\sin(\omega t)) \\ &\dot{\phi} = k\eta \\ &\dot{\eta} = -\omega_l \eta + \omega_l \Delta \sin(\omega t) \\ &\dot{\zeta} = \omega_h \Delta. \end{aligned} \quad (16)$$

Now, we introduce the new coordinates as

$$\tilde{\theta} = \theta - \theta^* \quad (17)$$

$$\tilde{\phi} = \phi - \phi^* \quad (18)$$

$$\tilde{\zeta} = \zeta - J(l(\phi^*)). \quad (19)$$

Then, in the time scale $\tau = \omega t$, the system (16) is rewritten as

$$\begin{aligned} &\omega\dot{z} = \bar{A}z + \bar{B}(\tilde{\theta} + \theta^*) \\ &\omega\dot{\tilde{\theta}} = \gamma(z, \tilde{\phi} + \phi^* + a\sin(\tau)) \end{aligned} \quad (20)$$

$$\begin{aligned} &\dot{\tilde{\phi}} = \delta K'\eta \\ &\dot{\eta} = \delta[-\omega'_L \eta + \omega'_L (J(z) \\ &\qquad - J(l(\phi^*)) - \tilde{\zeta}) \sin(\tau)] \\ &\dot{\tilde{\zeta}} = \delta[\omega'_H (J(z) - J(l(\phi^*)) - \tilde{\zeta})]. \end{aligned} \quad (21)$$

C. *Stability Analysis of State Feedback Linearizable Nonlinear Systems*

Stability analysis of systems is required to check the stability of the whole system dynamics including control dynamics.

The system (20) is in standard singular perturbation form, where the singular perturbation parameter is ω. To obtain the fast and slow systems, we set $\omega = 0$ and "freeze" z at its "equilibrium value"

$$z = l(\tilde{\phi} + \phi^* + a\sin(\tau)). \tag{22}$$

Now, we study the system (21) and substitute (22) into (21) to get the "reduced system"

$$\begin{aligned} \dot{\tilde{\phi}}_r &= \delta K' \eta_r \\ \dot{\eta}_r &= \delta[-\omega_L' \eta_r + \omega_L'(v(\tilde{\phi}_r + a\sin(\tau)) \\ &\quad - \tilde{\zeta}_r)\sin(\tau)] \\ \dot{\tilde{\zeta}}_r &= \delta[-\omega_H' \tilde{\zeta}_r + \omega_H' v(\tilde{\phi}_r + a\sin(\tau))]. \end{aligned} \tag{23}$$

where $v(\tilde{\phi}_r + a\sin(\tau)) = J(l(\tilde{\phi}_r + \phi^* + a\sin(\tau))) - J(l(\phi^*))$, and it must satisfy that

$$\begin{aligned} & v(0) = 0 \\ & v'(0) = J'(l(\phi))|_{\phi=\phi^*} = 0 \\ & v''(0) = J''(l(\phi))|_{\phi=\phi^*} < 0. \end{aligned} \tag{24}$$

In the study of dynamical systems, the method of averaging is used to study certain time-varying systems by analyzing easier, time-invariant systems obtained by averaging the original system.

The averaging model for every system with period T is

$$\dot{x}_{avg} = \frac{1}{T}\int_0^T f(\tau, x, 0)d\tau = \tilde{f}(x_{avg}). \tag{25}$$

Now, we apply averaging model form in (25) to the system (23), and we get

$$\begin{aligned} \dot{\tilde{\phi}}_r^{avg} &= \delta K' \eta_r^{avg} \\ \dot{\eta}_r^{avg} &= \delta[-\omega_L' \eta_r^{avg} + \frac{\omega_L'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} \\ &\quad + a\sin(\tau))\sin(\tau)d\tau] \\ \dot{\tilde{\zeta}}_r^{avg} &= \delta[-\omega_H' \tilde{\zeta}_r^{avg} + \frac{\omega_H'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} \\ &\quad + a\sin(\tau))d\tau]. \end{aligned} \tag{26}$$

Then, the average equilibrium $(\tilde{\phi}_r^{avg}, \eta_r^{avg}, \tilde{\zeta}_r^{avg})$ which satisfies

$$\eta_r^{avg} = 0, \tag{27}$$

$$\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))\sin(\tau)d\tau = 0, \tag{28}$$

$$\tilde{\zeta}_r^{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))d\tau. \tag{29}$$

By postulating $\tilde{\phi}_r^{avg}$ in the form

$$\tilde{\phi}_r^{avg} = b_1 a + b_2 a^2 + O(a^3), \tag{30}$$

substituting in (28), using (24), integrating, and equating the like power of a, we get $v''(0)b_1 = 0$ and $v''(0)b_2 + \frac{1}{8}v'''(0) = 0$, which implies that

$$\tilde{\phi}_r^{avg} = -\frac{v'''(0)}{8v''(0)}a^2 + O(a^3). \tag{31}$$

Also by using same calculations, we get

$$\tilde{\zeta}_r^{avg} = -\frac{v''(0)}{4}a^2 + O(a^3). \tag{32}$$

Thus, the equilibrium of the averaging model (26) is

$$\begin{aligned} \tilde{\phi}_r^{avg} &= -\frac{v'''(0)}{8v''(0)}a^2 + O(a^3) \\ \eta_r^{avg} &= 0 \\ \tilde{\zeta}_r^{avg} &= -\frac{v''(0)}{4}a^2 + O(a^3). \end{aligned} \tag{33}$$

Then, the Jacobian of (26) at $(\tilde{\phi}_r^{avg}, \eta_r^{avg}, \tilde{\zeta}_r^{avg})$ is

$$J_r^{avg} = \delta \begin{bmatrix} 0 & K' & 0 \\ \frac{\omega_L'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))\sin(\tau)d\tau & -\omega_L' & 0 \\ \frac{\omega_H'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))d\tau & 0 & -\omega_H' \end{bmatrix}. \tag{34}$$

Since J_r^{avg} is Hurwitz if and only if

$$\frac{\omega_L'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))\sin(\tau)d\tau < 0. \tag{35}$$

By using (24), we get

$$\frac{\omega_L'}{2\pi}\int_0^{2\pi} v(\tilde{\phi}_r^{avg} + a\sin(\tau))\sin(\tau)d\tau = \pi v''(0)a + O(a^2) \tag{36}$$

which is clear to see that it is less than zero if a is chosen sufficiently small. This implies that the equilibrium of the average system (26) is exponentially stable.

Theorem 2.3.1. Consider system (20) under *Assumption 2.2.3.* There exist $\bar{\bar{\delta}}$ and $\bar{\bar{a}}$ such that for all $\delta \in 0, \bar{\bar{\delta}}$ and $a \in \bar{\bar{a}}$ system (23) has a unique exponentially stable periodic solution $(\tilde{\phi}_r^{2\pi}(\tau), \eta_r^{2\pi}(\tau), \tilde{\zeta}_r^{2\pi}(\tau))$ of period 2π and this solution satisfies

$$\left|\begin{bmatrix}\tilde{\phi}_r^{2\pi}(\tau)+\frac{v'''(0)}{8v''(0)}a^2\\ \eta_r^{2\pi}(\tau)\\ \tilde{\zeta}_r^{2\pi}(\tau)-\frac{v''(0)}{4}a^2\end{bmatrix}\right|\le O(\delta)+O(a^3),\forall\tau\ge 0. \quad (37)$$

This result, along with the triangle inequality, implies that all solutions $(\tilde{\phi}_r(\tau),\eta_r(\tau),\tilde{\zeta}_r(\tau))$ converge to an $O(\delta+a^3)$-neighborhood of the origin. In other words, these solutions converge as close as possible to zero. It is important to interpret this result in terms of the system (21).

Since $y=J(l(\phi^*+\tilde{\phi}_r+a\sin(\tau)))$, we have

$$\begin{aligned}y&=J(l(\phi^*))+J'(l(\phi^*))(\tilde{\phi}_r)+a\sin(\tau))\\&+J''(l(\phi^*))(\tilde{\phi}_r+a\sin(\tau))^2\\&+O((\tilde{\phi}_r+a\sin(\tau))^3),\end{aligned} \quad (38)$$

where, $J'(l(\phi^*))=0,$ and then

$$\begin{aligned}y-J(l(\phi^*))&=J''(l(\phi^*))(\tilde{\phi}_r+a\sin(\tau))^2\\&+O((\tilde{\phi}_r+a\sin(\tau))^3),\end{aligned} \quad (39)$$

where,

$$\begin{aligned}\tilde{\phi}_r+a\sin(\tau)&=(\tilde{\phi}_r-\tilde{\phi}_r^{2\pi})+(\tilde{\phi}_r^{2\pi}+\frac{J'''(l(\phi^*))}{8J''(l(\phi^*))}a^2)\\&-\frac{J'''(l(\phi^*))}{8J''(l(\phi^*))}a^2+a\sin(\tau).\end{aligned} \quad (40)$$

Since the first term converges to zero, the second term is $O(\delta+a^3)$, the third term is $O(a^2)$ and the fourth term is $O(a)$, then we conclude

$$\begin{aligned}\tilde{\phi}_r+a\sin(\tau)&=O(a+\delta)\\ y-J(l(\phi^*))&=O(a^2+\delta^2).\end{aligned} \quad (41)$$

Now, we address the full system whose state space model is given by (20) and (21) in the time scale $\tau=\omega t$.

By Theorem 2.3.1, there exists an exponentially stable periodic solution $(\tilde{\phi}_r^{2\pi}(\tau),\eta_r^{2\pi}(\tau),\tilde{\zeta}_r^{2\pi}(\tau))$ such that

$$\begin{aligned}\dot{\tilde{\phi}}_r^{2\pi}&=\delta K'\eta_r^{2\pi}\\ \dot{\eta}_r^{2\pi}&=\delta[-\omega_L'\eta_r^{2\pi}+\omega_L'(v(\tilde{\phi}_r^{2\pi}\\&+a\sin(\tau))-\tilde{\zeta}_r^{2\pi})\sin(\tau)]\\ \dot{\tilde{\zeta}}_r^{2\pi}&=\delta[-\omega_H'\tilde{\zeta}_r^{2\pi}+\omega_H'v(\tilde{\phi}_r^{2\pi}\\&+a\sin(\tau))].\end{aligned} \quad (42)$$

To bring the system (20) and (21) into the standard singular perturbation form, we shift the state s where $s=(\tilde{\phi},\eta,\tilde{\zeta})$ using the transformation

$$\tilde{s}=s-s_r^{2\pi} \quad (43)$$

and then we get

$$\begin{aligned}\omega\dot{z}&=\bar{A}z+\bar{B}(\tilde{\theta}+\theta^*)\\ \omega\dot{\tilde{\theta}}&=\gamma(z,\tilde{s}_1+\tilde{\phi}_r^{2\pi}+\phi^*+a\sin(\tau))\end{aligned} \quad (44)$$

$$\dot{\tilde{s}}=\delta[G(\tau,z,\tilde{s}+s_r^{2\pi})-\bar{G}(\tau,s_r^{2\pi})]. \quad (45)$$

Then, we note that

$$z=L(\tau,\tilde{s}+s_r^{2\pi}) \quad (46)$$

is the quasi-steady state, and that the reduced model

$$\dot{\tilde{s}}_r=\delta\bar{G}(\tau,L(\tau,\tilde{s}+s_r^{2\pi}),\tilde{s}+s_r^{2\pi}) \quad (47)$$

has an equilibrium at the origin $\tilde{s}_r=0$. Also, to complete the singular perturbation analysis, we study the boundary layer model when $\tilde{z}=z-L(\tau,\tilde{s}+s_r^{2\pi})$, then

$$\begin{aligned}\dot{\tilde{\theta}}&=\frac{1}{\omega}\gamma(\tilde{z}+L(\tau,\tilde{s}+s_r^{2\pi}),\tilde{s}_1+\tilde{\phi}_r^{2\pi}\\&+\phi^*+a\sin(\tau))\\&=\frac{1}{\omega}\gamma(\tilde{z}+l(\hat{\phi}),\hat{\phi})\end{aligned} \quad (48)$$

where $\hat{\phi}=(\tilde{\phi}+\phi^*+a\sin(\tau))$. Since $\gamma(l(\hat{\phi}),\hat{\phi})\equiv 0$, then $\tilde{z}=0$ is an equilibrium of (48). By *Assumption 3.2.2*, this equilibrium is exponentially stable uniformly in $\hat{\phi}$.

Now, from the system (44.1) when $\tilde{z}=z-\bar{l}(\theta^*)$ which satisfies $z=\bar{l}(\theta)$ or $z=l(\hat{\phi})$ in *Assumption 2.2.2*, we get

$$\begin{aligned}\dot{\tilde{z}}&=\frac{1}{\omega}[\bar{A}\tilde{z}+\bar{A}\bar{l}(\theta^*)+\bar{B}\bar{\theta}+B\theta^*]\\&=\frac{1}{\omega}[\bar{A}\tilde{z}+\bar{B}\tilde{\theta}].\end{aligned} \quad (49)$$

where $\bar{A}\bar{l}(\theta^*)+B\theta^*=0.$ Then, we note that $\tilde{\theta}=\hat{l}(\tilde{z}),$ and since we proved that $\tilde{z}=0,$ then also $\tilde{\theta}=0$.

D. Example and Results

Consider the nonlinear system

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=x_1^2+u\\ y&=J(x)\end{aligned} \quad (50)$$

where the cost function after we apply the SFL system is

$$\begin{aligned}J(z)&=-(z_1+3z_2)^2+10(z_1+3z_2)+1\\&=-(\hat{\phi})^2+10(\hat{\phi})+1,\end{aligned} \quad (51)$$

and $\gamma(z,\hat{\phi})$ is picked as

$$\gamma(z,\hat{\phi}) = z_1 + 3z_2 - \hat{\phi} \\ = z_1 + 3z_2 - \phi - a\sin(\omega t). \quad (52)$$

where $\hat{\phi} = \phi^*$ is the maximizer of the cost function $J(l(\phi))$ that has a maximum at ϕ. The picked gains are $k_1 = 8$ and $k_2 = 6$. Here, we let the frequency perturbation $\omega = 0.11$, amplitude perturbation $\alpha = 0.05, k = 0.011, \omega_h = 0.011,$ and $\omega_l = 0.011$. Next figures show the results of the system and the controller design. The initial conditions are $z_1(0) = 1, z_2(0) = -1, \theta(0) = \eta(0) = \zeta(0) = 0.$

In this study, we investigate an alternative extremum seeking scheme for nonlinear plant. The proposed scheme utilizes an explicit structure information of the objective function that depends on the full states availability. However, it is assumed that the objective function is not available for measurement. Next figures show the result of this example.

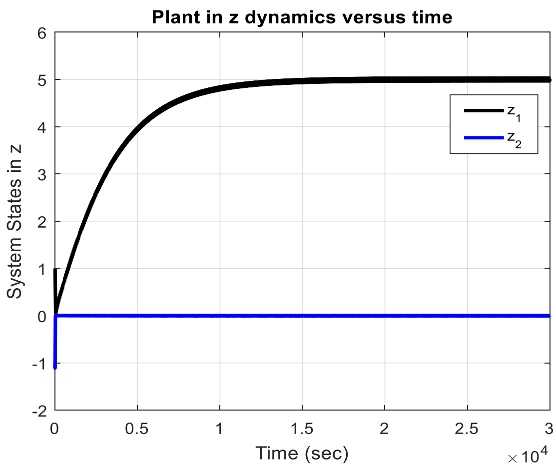
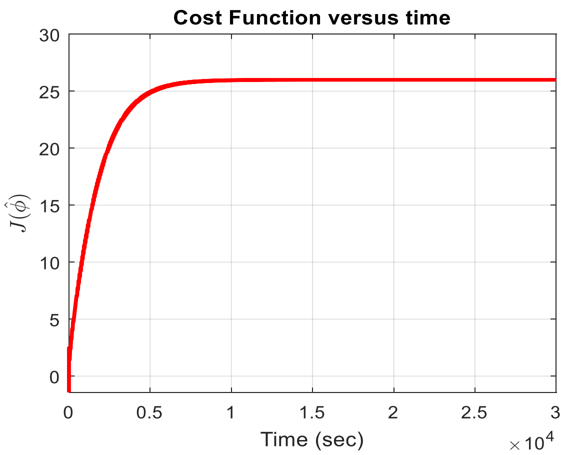


Figure. 1. Plot of the Plant States.



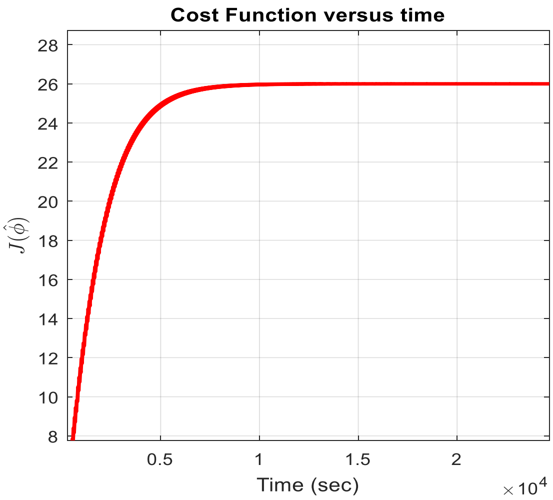


Fig. 2: Performance Function.

As shown in Figure. 1 that the plant states in z-dynamics, and in Figure. 2 that the cost function is driven to its maximum value which is $J = 26$.

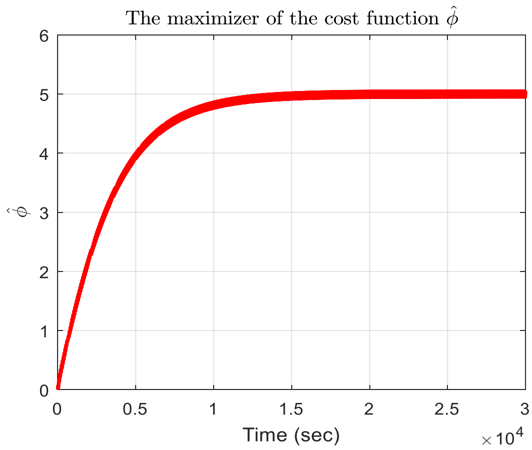


Figure. 3.Maximizer of the Cost Function $\hat{\phi}$.

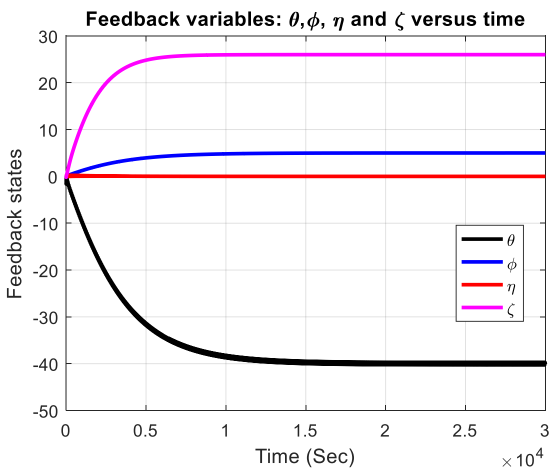


Figure. 4. Feedback Control Variables or States: θ, ϕ, η and ζ.

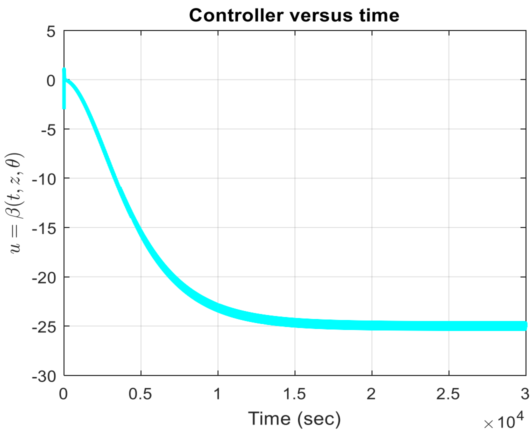


Figure. 5.Controller.

Also, we can see in Fig. 3 that the maximizer $\hat{\phi}$ is driven to value $\phi^* = 5$ that maximizes the cost function J. Fig. 4 shows that the feedback control variables are driven smoothly to their values, and Fig. 5 shows that the controller of the system is driven to *-25*.

III. OBSERVER OF SISO NONLINEAR SYSTEMS

In this section, we extend the work on (SFL) systems to be also input-output feedback linearizable (IOFL) systems. In this problem, we design a cost function for the mentioned system under some conditions, and we design a controller that lets the system's output drive the cost function to the minimum. The way to design the controller for SFL systems it is different from the design of the controller for both SFL and IOFL systems.

A. Problem Statement

Consider the SISO nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{53}$$

where $x \in \Re^n$ is the system states, $u \in \Re$ is the input, $f(.)$ and $g(.)$ are smooth functions and depend only on $x, y \in \Re$ is the output of the system of the state x at time t, and the output is measured by states and the assumption of full state availability is removed in the output.

The considered cost function

$$J = F(x) \tag{54}$$

is not actually the output of the system, and it is assumed as we mentioned in (2) and *Assumption 2.2.3* but in this case the cost function that we are maximizing is fed by the estimated states in the form $\hat{J} = F_0(\hat{x})$.

Assumptions 3.1.1. This assumption is the same as the assumption in (2.1.1), but we add here two more assumptions to the system as follows:

1- Input-Output feedback linearizable.
2- Linear output function.

Now by transferring the system (53) to the state feedback linearizable form, we get the same form in the system (4) which is

$$\begin{aligned} \dot{z} &= Az + B(\hat{f}(z) + \hat{g}(z)u) \\ &= Az + BG(z,u) \\ y &= h(z) = Cz \end{aligned} \tag{55}$$

where (*A;B*) is controllable canonical form which is

$$A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0_{1\times(n-1)} \end{bmatrix},$$

and $\hat{g}(z) = \dot{z} = \hat{g}(T(x))$ is non-singular for all x, and $G(z,u) = \hat{f}(z) + \hat{g}(z)u$ which is $G : \Re^n \to \Re^n$ is globally Lipschitz continuous and uniformly in t. In this part, we want to estimate the state z from the available signals u and y by applying the High Gain Observer (HGO) to the system (54). The reason of using the High Gain Observer (HGO) superior to others is that in the estimation of the variables of any system, the HGO gives very good estimation.

Here, we look for an “observer” of the state of the form

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + B(\hat{f}(\hat{z}) + \hat{g}(\hat{z})u) + H(y - \hat{y}) \\ &= A\hat{z} + BG_0(\hat{z},u) + H(y - \hat{y}) \\ y &= C\hat{z} \end{aligned} \tag{56}$$

where $H = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \end{bmatrix}^T$, and $G_0(\hat{z},u) = \hat{f}(\hat{z}) + \hat{g}(\hat{z})u$ is a nominal model of $G(z,u)$.

B. Perturbation-based Extremum Seeking Control Design for IOFL System

To design the PES controller, we have to deal with some errors dynamics. Some of the errors in this case of problem we already worked in *Section II*, and these errors should go as close as possible to zero when the time goes to infinity. The mean error in this section is the error between the really states and the observer states.

We analyze the high pass filter as we have done before but the difference here is that the HPF is multiplies by the estimated cost function. The errors that we have mentioned in *Section II* are the same, and we present here the following error.

Including to the errors that we worked on in the previous section, we added in this section an error that we deal with in this case which is $\tilde{e}(t) = z - \hat{z}$. This error leads us to the error dynamics

$$\begin{aligned} \dot{\tilde{e}} &= \dot{z} - \dot{\hat{z}} \\ \dot{\tilde{e}} &= (A - HC)\tilde{e} + B[G(z,u) - G_0(\hat{z},u)] \\ \dot{\tilde{e}} &= (A - HC)\tilde{e} + BL(z,\hat{z},u), \end{aligned} \tag{57}$$

where ($L(z,\hat{z},u) = G(z,u) - G_0(\hat{z},u)$. Note that it is possible to modify the dynamics (eigenvalues) of the error system by proper selection of the gain *H*. If the system is observable as we assume, it is always possible to find an observer gain *H* to set the eigenvalues of the error dynamics at arbitrary values, and we want to design *H* such that $\lim \tilde{e} = 0$ when $t \to \infty$.

In the absence of $L(z,\hat{z},u)$ or $G = G_0 = u$ in (55) and (56), asymptotic error convergence is achieved by designing H such that $A_0 = (A - HC)$ is *Hurwitz* (i.e. all its eigenvalues are on the left-half complex plane). In the presence of $L(z,\hat{z},u)$, we need to design H with the goal of rejecting the effect of $L(z,\hat{z},u)$ on $\tilde{e}$.

This is ideally achieved, for any $L(z,\hat{z},u)$, if the transfer function $G(s)$ from $L(z,\hat{z},u)$ to $\tilde{e}$ is ideally zero. This is impossible, but we can make $\sup|G(j\omega)|$ arbitrarily small by choosing $h_n >> h_{n-1} >> h_{n-2} >> \cdots >> 1.$

So, we can let $h_1 = \frac{\alpha_1}{\varepsilon}, h_2 = \frac{\alpha_2}{\varepsilon^2}, \cdots, h_n = \frac{\alpha_n}{\varepsilon^n}$ for some positive constants $\alpha_1, \alpha_2, \cdots, \alpha_n > 0$, and with ε arbitrarily small constant $0 < \varepsilon << 1$. This should satisfy that $\lim G(s) = 0$ when $\varepsilon \to 0$.

The observer eigenvalue is λ_i / ε , where λ_i , $i = 1, 2, \cdots, n,$, is the roots of

$$\lambda^n + \alpha_1 \lambda^{n-1} + \cdots + \alpha_n = 0$$
$$\sup \| G(j\omega) \| = O(\varepsilon)$$

In other words, the eigenvalues of the observer are assigned at $1/\varepsilon$ times the roots of the polynomial $s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_{n-1} s + \alpha_n$. Therefore, choosing ε sufficiently small lets the observer dynamics much faster than the closed-loop system under the state feedback system in (55).

Now, we introduce another estimation error by letting $\tilde{\eta} = Q(\varepsilon, \tilde{e})$ where $\tilde{\eta}_i = \frac{\tilde{e}_i}{\varepsilon^{n-i}}$, and $i = 1, 2, \cdots, n$. Then the newly defined variables satisfy the singularly perturbed equation, and the error dynamics becomes

$$\varepsilon \dot{\tilde{\eta}} = (A - HC)\tilde{\eta} + \varepsilon BL(z,\hat{z},u) \tag{58}$$

This equation shows clearly that reducing ε diminishes the effect of $L(z,\hat{z},u)$ and makes the dynamics of $\tilde{\eta}$ much faster than those of *z*. However, the scaling $\tilde{\eta}_i = \frac{\tilde{e}_i}{\varepsilon^{n-i}}$ shows that the transient response of the high gain observer suffers from a peaking phenomenon.

Now, we keep getting the derivative of the observer output $\hat{y} = h(\hat{z})$ until the controller must not vanish at *n*-derivative of the dynamics as

$$\overset{n}{\hat{y}} = \chi(z,\hat{z}) + g(\hat{z})u. \tag{59}$$

which can be globally stabilized by the state feedback controller or output controller. In other words, the system must has relative degree *n* where the relative degree *N* = *n* + *d* where *d = 0*. Then, the controller will be taken as

$$u = \beta(z,\hat{z},\theta) = \frac{-1}{g(\hat{z})}[\chi(z,\hat{z}) + \overline{v}(\hat{z}) + \theta], \tag{60}$$

where $g(\hat{z}) \neq 0$ for all $\hat{z}$, and $\overline{v}(\hat{z}) = k_1\hat{z}_1 + k_2\hat{z}_2 + \cdots k_n\hat{z}_n$. The dynamics of the system including the feedback system will be

$$\begin{aligned} \dot{z} &= Az + B(z,u) \\ \dot{\hat{z}} &= A\hat{z} + BG_0(\hat{z},u) + H(y - \hat{y}) \\ \dot{\theta} &= \gamma(\hat{z}, \phi + a\sin(\omega t)) \\ \dot{\phi} &= k\eta \\ \dot{\eta} &= -\omega_l \eta + \omega_l \Delta \sin(\omega t) \\ \dot{\zeta} &= \omega_h \Delta. \end{aligned} \tag{61}$$

Then, in the time scale $\tau = \omega t$, the system (61) is rewritten in new coordinates, and we keep the coordinates that we have worked on in *Section II* in (17), (18), and (19) as

$$\begin{aligned} \omega \frac{dz}{d\tau} &= Az + B(z,u) \\ \frac{d\hat{z}}{d\tau} &= A\hat{z} + BG_0(\hat{z},u) + H(y - \hat{y}) \\ \frac{d\theta}{d\tau} &= \gamma(\hat{z}, \phi + a\sin(\tau)) \end{aligned} \tag{62}$$

$$\begin{aligned} \frac{d\tilde{\phi}}{d\tau} &= \delta K'\eta \\ \frac{d\eta}{d\tau} &= \delta[-\omega_L'\eta + \omega_L' J(\hat{z}) \\ &\quad - J(l(\phi^*)) - \tilde{\zeta})\sin(\tau)] \\ \frac{d\tilde{\zeta}}{d\tau} &= \delta[\omega_H'(J(\hat{z}) - J(l(\phi^*)) - \tilde{\zeta})]. \end{aligned} \tag{63}$$

Now, we work on the error dynamics of $\tilde{\eta}$ in (58) in the mentioned time scale, and we keep (17), (18), and (19) the same because they lead us to the same results.

Then, the new observer error will be we get

$$\omega\varepsilon \frac{d\tilde{\eta}}{d\tau} = (A - HC)\tilde{\eta} + \varepsilon BL(z,\hat{z},u) = A_0\tilde{\eta} + \varepsilon BL(z,\hat{z},u) \quad (64)$$

where A_0 is stable and its eigenvalues are located on the left-half complex plane which means that $A_0 < 0$.

C. *Stability Analysis of Input-Output Feedback Linearizable Nonliear Systems*

In this section, we study the error dynamic in (64) which is seen clearly that reducing ε decreases the effect of $L(z,\hat{z},u)$ .In the behavior of the peaking phenomenon, it is clearly to see that $\eta(0)$ will be in $O(1/\omega\varepsilon)$ whenever $z(0) \neq \hat{z}(0)$. Then, the solution of $\tilde{\eta}(t)$ will have the term $(1/\omega\varepsilon)e^{-1t/\omega\varepsilon}$ for some $a > 0$. Although this exponential mode decays rapidly, it exhibits an impulsive-like behavior where the transient peaks to the neighborhood of $1/\omega\varepsilon$ values before it decays rapidly towards zero.

In fact, $(a/\omega\varepsilon)e^{-1t/\omega\varepsilon}$ approaches an impulse function as $\varepsilon \to 0$. In the peaking phenomenon method, the controller can be driven out of its region of attraction, thereby finally causing instability.

We still can get good results as in the example results in *Section II.D* later, but that does not always guaranteed. To solve the problem of the *peaking phenomenon*, we know that this phenomenon is an artifact of the high-gain observer. Therefore, we should disregard the large, unrealistic values of the state estimate. To do so, we can design the function $G_0(\hat{z},u)$ to be globally bounded in $\hat{z}$. For the control law $u(t)$, it will be bounded by applying the saturation function. Therefore, this boundedness can be always achieved by saturating $u(t)$ outside compact region of interest so that $u(t)$ is a globally bounded function. Then, the destabilizing effect by adapting the saturation approach will be reduced. The saturating of $u(t)$ and the global boundedness of $G_0(\hat{z},u)$ in $\hat{z}$ provides a buffer that protects the plant from peaking because during the peaking period, the control $u(t)$ saturates. Because the peaking period shrinks to zero as ε tends to zero, for sufficiently small ε, the peaking period is so small that the state of the plant z remains close to its initial value. Consequently, the trajectories of the closed-loop system under the output feedback controller approach its trajectories under the state feedback controller as ε tends to zero. This leads to recovery of the performance achieved under state feedback [13,14].

Let $V(z)$ be a Lyapunov function for the slow subsystem which is guaranteed to exist for any stabilizing state feedback control by the converse Lyapunov theorem. Let $W(\tilde{\eta}_{avg}) = \varepsilon\tilde{\eta}_{avg}^T P_0 \tilde{\eta}_{avg}$ be a Lyapunov function for the fast subsystem, where $P_0 = P_0^T$ is the positive definite solution of the Lyapunov equation $P_0A_0 + A_0^TP_0 = -I$ where $A_0 = A - HC$, and this can be proved by repeating the arguments in [14].

Now, we analysis the stability to study the system (63) as we have done to the system (21) while we "freeze" here $\hat{z}$ in (62) at its equilibrium value

$$\begin{aligned} \hat{z}_{2,\cdots,n} &= f_1(\bar{\eta}_1) \\ \hat{z}_1 &= f_2(f_1(\bar{\eta}_1), \tilde{\phi} + \phi^* + a\sin(\tau)) \end{aligned} \quad (65)$$

and substitute it into (63), and we get the same reduced system in (23) where the only difference between the system (21) and the system (64) is that the system (63) is fed by the function $\hat{J}(\hat{z})$ instead of $J(z)$. The errors proof of the system (63) is the same as in (21).

From (41), it is clear to see the same results here while we have the same assumption for $\hat{J}$ as we have with J. From the error $\tilde{\hat{z}} = \tilde{\hat{z}} - \aleph(f_1, f_2)$, and then its dynamics will be

$$\dot{\tilde{\hat{z}}} = \aleph_0(\tilde{\hat{z}}, \tilde{\phi}, \bar{\eta}_1). \quad (66)$$

From (66), it is clear to see that the equilibrum of $\tilde{\hat{z}} = \aleph_1(\tilde{\phi}, \bar{\eta}_1)$ where $\tilde{\phi} = O(a+\delta)$ and $\bar{\eta}_1 = O(\varepsilon)$. Also, we study the error of the second equation in (62) which is $\tilde{\hat{z}} = \hat{z} - \aleph_1(\bar{\eta}, \theta)$, so the dynamics of this error will be

$$\dot{\tilde{\hat{z}}} = \aleph_2(\tilde{\hat{z}} + \aleph_1(\bar{\eta}, \tilde{\theta} + \theta^*), \bar{\eta}, \tilde{\theta} + \theta^*). \quad (67)$$

Then, we conclude that $\tilde{\theta}$ is the solution of this equation which is $\tilde{\theta} = \aleph_3(\tilde{\hat{z}}, \bar{\eta})$ where $\tilde{\hat{z}}$ and $\bar{\eta}$ are near to zero.

Also, we study the error of the first equation in (62) which is described as $\tilde{z} = z - \aleph_4(\hat{z}, \theta, \bar{\eta})$, and then the dynamics of this error will be

$$\begin{aligned} \dot{\tilde{z}} &= \aleph_5(z + \bar{\eta}, \tilde{\theta} + \theta^*, \bar{\eta}) \\ &= \aleph_5(\tilde{z} + \aleph_4(\tilde{z}, \theta, \bar{\eta}) + \bar{\eta}, \tilde{\theta} + \theta^*, \bar{\eta}) \\ &= \aleph_5(\tilde{z} + \aleph_4(\tilde{\hat{z}} + \aleph(f_1, f_2), \tilde{\theta} + \theta^*, \bar{\eta}) \\ &\quad + \bar{\eta}, \tilde{\theta} + \theta^*, \bar{\eta}) \end{aligned} . \quad (68)$$

From (68), we conclude that the equilibrum is $\tilde{z} = \aleph_6(\tilde{\hat{z}}, \bar{\eta}, \tilde{\theta}) \approx O(0)$ where $\tilde{\hat{z}}, \bar{\eta}$ and $\tilde{\theta}$ are very close to zero.

Now, we conclude that all errors $(\tilde{z}, \tilde{\hat{z}}, \bar{\eta}, \tilde{\theta}, \tilde{\phi}, \tilde{\zeta})$ go as close as possible to zero. In the observer part, we investigate an alternative perturbation-based extremum seeking scheme for the observer of nonlinear plant under the given assumptions. The proposed scheme utilizes an explicit structure information of the objective function that depends on the observer output plant and a new state from the controller. However, it is assumed that the objective function is not available for measurement.

D. *Examples and Results*

Consider the SISO nonlinear system (1) as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + u \\ y &= x_1 \\ h &= J(x) \end{aligned} \quad (69)$$

where $J(x)$ is the cost function which is estimated as

$$\begin{aligned} \hat{J}(\hat{z}) &= -(\hat{z}_1^2 + 3\hat{z}_2)^2 + 2(\hat{z}_1^2 + 3\hat{z}_2) + 1 \\ &= -(\hat{\phi})^2 + 2(\hat{\phi}) + 1, \end{aligned} \quad (70)$$

and $\gamma(\hat{z}, \hat{\phi})$ is picked as

$$\begin{aligned} \gamma(\hat{z}, \hat{\phi}) &= -\hat{z}_1^2 + 3\hat{z}_2 - \hat{\phi} \\ &= \hat{z}_1^2 + 3\hat{z}_2 - \phi - a\sin(\omega t). \end{aligned} \quad (71)$$

where $\hat{\phi} = \phi^*$ is the maximizer of the cost function $\hat{J}(l(\hat{\phi}))$ that has a maximum at $\hat{\phi}$.

Here, we let the frequency perturbation $\omega = 0.2$, amplitude perturbation $\alpha = 0.05, k = 0.1, \omega_l = 0.1$, and $\omega_h = 0.1$. The high gain observer gains are; $\alpha_1 = \alpha_2 = 1$ and the variable $\varepsilon = 0.01$ with initial conditions $z_1(0) = 0.1, z_2(0) = -0.2, \hat{z}_1(0) = 0.11$, and $\hat{z}_2(0) = -0.22$. Also, $\theta(0) = \phi(0) = \zeta(0) = 0$. Next figures show the results of the system and the controller design.

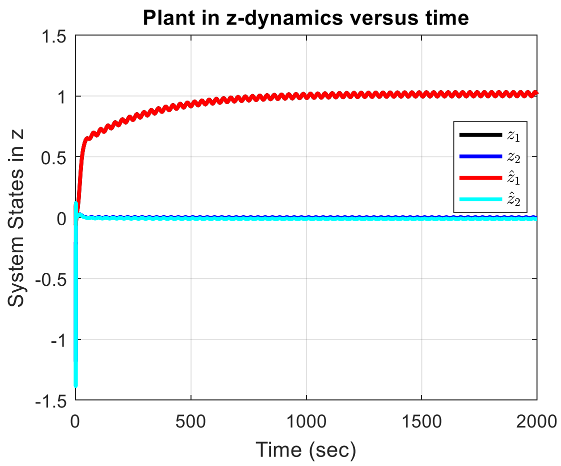


Figure. 6. Plot of Real and Estimated States of SFL System.

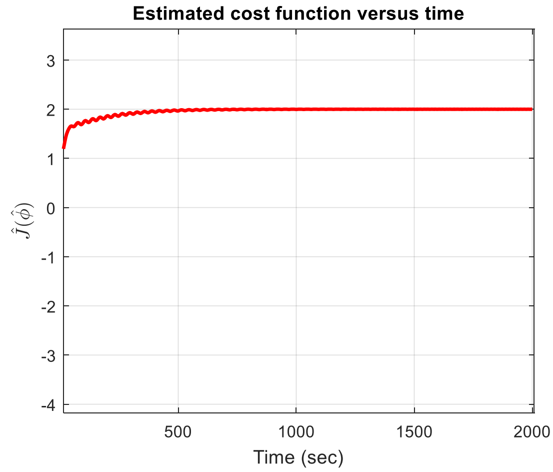


Figure. 7.Estimated Performance Function $\hat{J}(\hat{z})$.

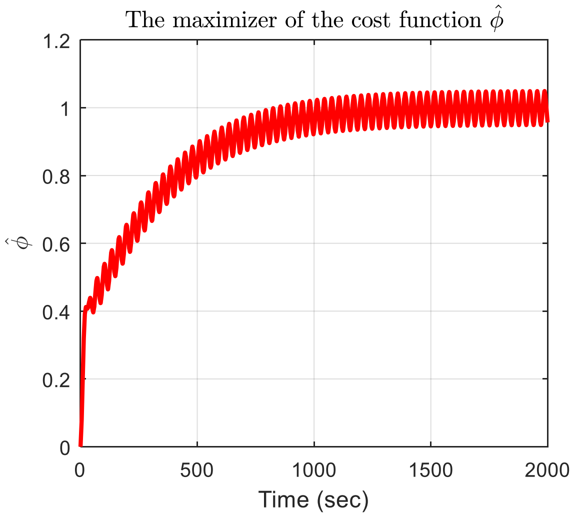


Figure. 8.Maximizer of the Cost Function $\hat{\phi}$.

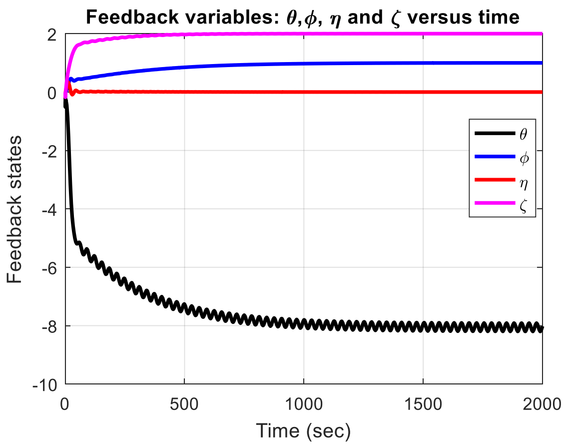


Figure. 9. Feedback control Variables or States: θ, ϕ, η and ζ.

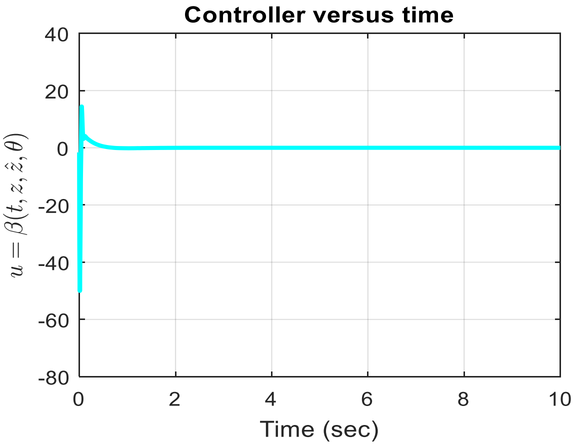


Figure. 10. PES Controller.

As shown in Fig. 6, the observer is working very well by tracking the real states (z_1, z_2) to the estimated states $(\hat{z}_1, \hat{z}_2)$, and as we see that the error of tracking is very small and barely we see the difference. They track each other in very short time because her we assume that their initial conditions are very close. It is clear that the real states are driven or tracked to the estimated states in very short time. Also, in Fig. 7 it is shown that the estimated cost function is driven to its maximum value $\hat{J} = 2$ by the maximizer $\hat{\phi} = 1$ which is shown in Fig. 8. Again, in Fig. 6, the small ε the fast tracking between z and $\hat{z}$. The feedback control variables; θ, ϕ, η and ζ are driven to their values smoothly as in Fig. 9. The controller result is going to zero as in Fig. 10.

IV. CONCLUSION

The control approach, PESC, solves a new performance (cost) function problem for driving this function to the maximum value and keeps the system in stable case. A stability analysis technique is used to approximate the unknown function and to steer the system to its unknown extremum. From the experience of extremum seeking control design, the new proposal based dynamic control approach is easy to understand and not easy to implement, which makes it quite practical. Our proof covers only one implementation of extremum control which is the method with a periodic perturbation. This approach is applied to the observer of the Single-Input Single-Output (SISO) nonlinear systems so that the performance function can reach its maximum value. The tow applied controllers take care of the maximizing of the cost function. From all of the obtained results, we can say that this controller (PESC) is working very well under the given conditions and with certain initial conditions. As a future work, the extremum seeking control design for the observer SISO discrete-time nonlinear systems may be investigated.

REFERENCES

[1] J. Sternby, "Extremum control systems: An area for adaptive control," in Preprints of the Joint American Control Conference, San Francisco, CA, 1980.

[2] K. J. Astrom and B. Wittenmark, Adaptive Control , 2nd ed. Addison-Wesley, Boston, MI, 1995.

[3] M. Leblanc, "Sur l'electrification des chemins de fer au moyen de courants alternatifs de frequence elevee," Revue Generale de l'Electricite, 1922.

[4] I. I. Ostrovskii, "Extremum regulation," Auto- matic and Remote Control, vol. 18, pp. 900{907, 1957.

[5] C. S. Drapper and Y. T. Li, Principles of Optimal- izing Control Systems and an Application to the Internal Combustion Engine. SME Publications, 1951.

[6] P. F. Blackman, "Extremum-seeking regulators," in An exposition of adaptive control, J. H. Westcott, Ed. New York, NY: The Macmillan Company, 1962.

[7] P. F. Blackman, "Design of a single-input sinusoidal-perturbation extremum-control system," in Proc. Inst. Elect. Eng, vol. 115, 1968, pp. 212-217.

[8] M. Guay D. Dochain M. Perrier, "ADAPTIVE EXTREMUM SEEKING CONTROL OF CONTINUOUS STIRRED TANK BIO- EACTORS," Work supported by the Natural Sciences and Engineering Research Council of Canada. University, Kingston.

[9] Lina Fu, "MODEL-BASED EXTREMUM SEEKING CONTROL FOR A CLASS OF NONLINEAR SYSTEMS," Dissertation Work. Ohio State University, 2010.

[10] M. Krstic, "Performance improvement and limitations in extremum seeking control," Systems and Control Letters, vol. 39, no. 5, pp. 313-326, 2000.

[11] Chunlei Zhang, Raul Ordonez, "Extremum- Seeking Control and Applications, " Dept. Electrical and Computer Engineering, University of Dayton. SBN: 978-1-4471-2223-4. Springer-Verlag London Limited, 2012.

[12] Miroslaz Krstic, Hsin-Hsiung Wang. "Stability of extremum seeking feedback for general nonlinear dynamic systems," PERGAMON, Automatica 36 (2000) 595-601.

[13] Yaodong Pan and Ozguner, U. "Sliding Mode Extremum Seeking Control for Linear Quadratic Dynamic Game," IEEE, Boston, MA, USA. ISSN:0743-1619. June 30 2004-July 2 2004.

[14] A. N. Atassi and H. K. Khalil, "A Separation Principle for the Stabilization of a Class of Nonlinear Systems," IEEE Trans. on Aut. Cont., vol. 44, no. 9, pp. 1672-1687, 1999.

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