Mathematical Model for Planning and Optimisation of Petroleum Supply Chain Under Uncertainty

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Abstract—**This work focuses on the planning and petroleum industry logistic and supply chains from raw materials to production distribution by developing a two-stage stochastic linear programming with recourse techniques. The paper investigates and creates a set of mathematical models which considers different parameters such as production of crude oil, transportation plan, production levels, operating conditions, production distribution plans, prices of raw materials and products under significant source of uncertainty which represent the market demand of refinery products. Expected Value of Perfect Information (EVPI) is computed a maximum amount a decision maker that should pay for additional information gives a perfect signal as to the state of nature.**

Index Terms: **petroleum supply chain, logistics, optimisation, uncertainty, stochastic programming**

I. INTRODUCTION

A supply chain defined as an integrated process wherein a number of various business entities (i.e., suppliers, manufacturers, distributors, and retailers) work together to convert raw materials into valuable final products and deliver these products to customers. Supply chain design and planning determine the optimal solutions to use all activities and entities (production, inventory and distribution) resources in the chain to meet market demand forecast in an economically efficient manner.

A typical petroleum supply chain involves oil exploration, oil production, oil transportation, crude oil storage tanks (which is connected with refinery by pipeline network), refinery operations, finished products inventory and distribution centres. All levels of decisions arise in such a supply chain namely, strategic, tactical and operational.

The high degree of uncertainty is an extremely important factor characterizing for planning and scheduling problems of the process industries.

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The petroleum industry is subject to a number of uncertainties such as a fluctuation of raw materials and refined product prices, variable reserves and production and size of market demand. (Lababidi, Ahmed, Alatiqi & Al-Enzi, 2004) introduced the uncertainties in demand, market prices, raw material costs and production yields. They found that uncertainties have a dramatic effect on the planning decision of the petrochemical supply chain and the most important parameter was the market's demand [1]. "Figure. 1" shows the market uncertainty in supply chain. [2].

Figure 1. The Market Uncertainty in Supply Chain

(Al-Othman, et al., 2008) studied the effect of uncertainties in market prices and market demand on the supply chain of petroleum organization owned by producing country the conclusion showed that uncertainty in the market demand has more impact on the supply chain plan than the actual market prices [3].

Supply chain design and planning determine the optimal solutions to use all activities and entities (production, inventory and distribution) resources in the chain to meet market demand forecast in an economically efficient manner.

Planning problems can be categorized as strategic, tactical or operational depending on the decision involved and the time horizon under consideration (Grossmann, et al., 2002) [4]. Strategic planning covers the longest time horizons in the range of one to several years and decisions cover the whole width of the organization, while focussing on major investments. Tactical planning typically covers the midterm horizon of between a few months to a year and decisions cover issues such as production, inventory and distribution. Production supply chain planning is a good example of tactical planning; e.g. (McDonald & Karimi, 1997) [5]; (Perea, et al., 2000) Operational planning usually covers a horizon of one week to three months and involves decisions regarding the actual operations and resource allocation [6]. Application includes the operational planning of utility system e.g. (Lyer & Grossmann, 1998) [7] and planning of refinery operation (Moro & Pinto, 1998) [8].

Mathematical Programming techniques have been developed and applied since the late 1940s. Dantzig (1947) invented and developed the simplex algorithm and really created the area of linear programming (LP) [9]. Stochastic linear programming with recourse was introduced by Dantzig and Beale in 1950's, as a mathematical programming technique for dealing with uncertainty (Valdimirou & Zenios, 1997) [10]. The fundamental idea behind stochastic linear programming is the concept of recourse. Recourse is the ability to take corrective action after a random event has taken place. Recourse programs are those programs in which some decisions are a recourse action that can be taken once uncertainty is disclosed.

In two-stage recourse models, the decision variables are classified according to whether they are implemented before or after observing of an outcome of a random variable. Decisions that are implemented before the actual realization of random parameters are known as first-stage decisions. Once the uncertain events have presented themselves, further design or operational adjustments can be made through values of the second-stage or alternatively called recourse variables at a particular cost. Due to uncertainty, the second-stage cost is a random variable. The objective is to choose the first-stage variables in a way that the sum of the first-stage costs and the expected value of the random second-stage costs is minimised. The concept of recourse has been applied to a linear, integer, and non-linear programming (Sahinidis, 2004) [11]. The considered two-stage problem is a linear because the objective functions and the constraints are linear.

II. PETROLEUM SUPPLY CHAIN NETWORK

The petroleum supply chain proposed in this work is illustrated in “Figure. 2.” It includes majority of the activities related to raw materials supply to a final product passing through a complex logistics network including warehouse, transportation, distribution centres and several conversion processes that take place in refinery plant.

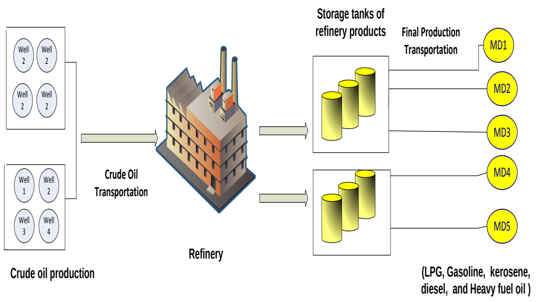

Figure 2. The Proposed Petroleum Supply Chain Network

The proposed network of petroleum supply chain is designed to start from crude oil production which is considered the first variable of supply chain model. The amount of crude oil transported from production sites to the refinery is the second variable of the model of the proposed petroleum supply chain.

The oil refinery activity is considered one of the most complex activities in the petroleum industry since it carries different processes to transform crude oil into valuable refined products of higher aggregate value, in addition to maximizing the profit.

The oil refinery essentially involves two categories of processes: physical and chemical processes, the physical separation processes of crude oil into a range of homogeneous petroleum fractions. In distillation unit, crude oil is entering the refinery undergoes primary separation by continuous atmospheric distillation to yield a variety of homogeneous fraction boiling over a wide range. A number of refinery products from distillation units such as LGP, Gasoline, Kerosene, Diesel and Heavy fuel oil are shipped to demand sources immediately. Chemical conversion processes of certain fractions to alter the product yield and improve product quality. The refineries produce light fraction products such as naphtha, gasoline, LPG (liquid petroleum gas) and propylene, medium products as (aviation kerosene and diesel) and heavy fractions such as (paraffin, lubricants, light crude oil, gas oil, coke and fuel oil). The volume of refinery production is one of the variables accounted in the proposed model. The storage capacity of the final product has also been considered in the model as it is a significant variable and has an effect on the supply chain optimisation. The quantities of refinery products that shipped to distribution centres, quantities of backlog, shortage demand have been taken into account.

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The problem is to develop an optimisation model for the planning of petroleum supply chain mentioned above that accounts for time periods of one year. Decisions related to production quantities of crude oil, transportation plan, storage capacities, production quantities of refinery, shortage demand, backlog and amount of final production shipped are needed for the planning purpose.

Choosing the best configuration for a petroleum supply chain and the ideal design and plan for all activities and entities of the supply chain are difficult tasks due to the high number of variables and constraints in a model. Mathematical programming plays a crucial role in solving such a problem, assisting in the decision-making process and in the planning of all activities at the strategic level.

III. MATHEMATICAL MODEL

A. Standard form

A standard formulation of the two-stage stochastic linear program with recourse is (Birge and Louveaux, 1997):

$$Min_x \;\; C^T x + E_\xi [\min q(\omega)^T y(\omega)]$$

s.t. $\;Ax = b$

$$T(\omega)x + Wy(\omega) = h(\omega)$$

$$x \geq 0 \;\; , \;\; y(\omega) \geq 0$$

Where

$x \in \mathcal{R}^{n1}$ represents the vector of first-stage decision variables (to be determined)

$c \in \mathcal{R}^{n1}$ vectors of (known) coefficients

A is an $m \; x \; n$ constraint matrix

$b \in \mathcal{R}^{m1}$ right hand vector.

E_ω expectation probability of occurrence of different scenarios.

$\omega \in \Omega$ outcomes of a experiment.

Ω set of all outcomes of a random experiment.

$y \in \mathcal{R}^{n2}$ represents the vector of second-stage decision variables.

$q \in \mathcal{R}^{n2}$ second stage-decision vector.

ξ a random vector whose realisation provides information on the second-stage decision y.

$h \in \mathcal{R}^{m2}$ a fixed vector

$W \in \mathcal{R}^{m2 \times n2}$ is a fixed matrix called recourse matrix.

$T \in \mathcal{R}^{m2 \times n1}$ is a random matrix with realisation called technology matrix.

First-stage decisions are represented by the vector x, while second-stage decisions are random events represented by the vector $y(\omega)$. The objective function in equation (1) contains a deterministic term $C^T x$ and the expectation of the second-stage objective $q(\omega)^T y(\omega)$ taken over all the realizations of the random event ω. For each ω, the value of $y(\omega)$ is the solution of linear programming. The first constraint is for the deterministic problem, while the second constraint is defined for each realisation, and the function of the random events as well as the first-stage variables.

B. Deterministic mathematical model

The proposed deterministic model addresses the portfolio optimization problem in the integration of an oil supply chain in order to satisfy the market demand with the lowest cost.

The planning of a petroleum supply chain proposed at the strategic level, and the planning horizon (T) for one year are assumed. The planning horizon is usually divided into time periods at which items of the plan are scheduled.

C. The objective function

The objective function of the proposed mathematical model is to optimise the petroleum resources by minimising the total costs of raw materials production, refinery and petrochemical production, raw material and final products transport, storage of final products, and penalty of the amount of shortage and backlog products for demand source as well as maximising the sale revenues.

The objective function for the deterministic model is defined as:

$$Z = \min\{[Production\ cost\ of\ crude\ oil] + [\,Production\ cost\ of\ final\ product] + [Transportation\ cost\ of\ crude\ oil] + [Stoarge\ cost] + [Transportation\ cost\ of\ final\ product] + [Penalty\ of\ shortage\ products] + [Backlog\ penalty\ of\ product] - [\,Sale\ revenue]\}$$

$$Z = \min\left\{\left[\sum_{i\in I}\sum_{t\in T} CO_i \,.\, Q_{i,t}\right] + \left[\sum_{j\in J}\sum_{t\in T} C_j \,.\, V_{j,t}\right] + \left[\sum_{i\in I}\sum_{t\in T} TC_i \,.\, TV_{i,t}\right] + \left[\sum_{j\in J}\sum_{t\in T} CS_j \,.\, SV_{j,t}\right] + \left[\sum_{j\in J}\sum_{md\in MD}\sum_{t\in T} CT_j \,.\, F_{j,md,t}\right] + \left[\sum_{j\in J}\sum_{md\in MD}\sum_{t\in T} \beta_{j,md} \,.\, DS_{j,md,t}\right] + \left[\sum_{j\in J}\sum_{md\in MD}\sum_{t\in T} CB_{j,md} \,.\, VB_{j,md,t}\right] - \left[\sum_{j\in J}\sum_{md\in MD}\sum_{t\in T} PS_{j,md} \,.\, F_{j,md,t}\right]\right\} \quad (1)$$

D. Constraints

Material balance for the final products:

$$\sum SV_{j,t-1} + V_{j,t} = \sum F_{j,md,t} + \sum SV_{j,t}$$
$$\forall_j \in J \;, md \in MD, t \in T \quad (2)$$

Crude oil constraint:

$$Q_{i,t} \leq Cp_{i,t} \; \forall_i \in I \;,\; t \in T \quad (3)$$

$$F_{j,d,t} \leq D_{jm,d,t} \qquad \forall_j \in J, d \in MD,\ t \in T \tag{4}$$

$$VB_{j,t-1} + D_{j,md,t} = \sum F_{j,md,t} + DS_{j,md,t} + B_{j,md,t}$$
$$\forall_j \in J, md \in MD,\ t \in T \tag{5}$$

$$DS_{j,md,t} = \delta \left(VB_{j,t-1} + D_{j,md,t} - \sum F_{j,md,t} \right)$$
$$\forall_j \in J, md \in MD,\ t \in T \tag{6}$$

$$\sum VB_{jmd,t} \leq \sum(\lambda V_{j,t})\ \forall_j \in J, md \in MD,\ t \in T \tag{7}$$

$$\sum DS_{jmd,t} \leq \sum(\delta D_{j,md,t})\ \forall_j \in J, md \in MD,\ t \in T \tag{8}$$

$$\sum SV_{j,t} \leq \sum SV_{j,t}^{max} \qquad \forall_j \in J,\ t \in T \tag{9}$$

$$TV_{i,t} \geq Q_{i,t} \qquad \forall_i \in I,\ t \in T \tag{10}$$

$$F_{j,d,t} \leq TP_{j,t}^{max} \qquad \forall_j \in J,\ t \in T \tag{11}$$

Production yield is defined as the final products that may be produced from processing crude oil:

$$\sum V_{j,t} = \sum(\gamma\, Q_{j,t}) \qquad \forall_j \in J,\ t \in T \tag{12}$$

E. Stochastic mathematical model

The formulation of two-stage stochastic linear program is:

$$Min_x\ C^T x + E_\xi [\min q(\omega)^T y(\omega)] \tag{13}$$

s.t. $Ax = b$

$$T(\omega)x + Wy(\omega) = h(\omega)$$

$$x \geq 0,\quad y(\omega) \geq 0$$

The objective function in equation (14) includes first-stage decision (deterministic term) $C^T x$ which represented by vector x, and expectation of the second-stage objective, $q(\omega)^T y(\omega)$ taken over all realization of the random events $\omega \in \Omega$, that represented by the vector $y(\omega)$.

For the petroleum supply chain optimisation problem presented in this study, the deterministic term corresponds to the crude oil quantity $Q_{i,t}$ and production volume $V_{i,t}$, during the planning horizon (T).The second-stage decision variables are represented by remaining terms for different scenarios.

In this section, the source of uncertainty in market demand for the final product of refineries and petrochemicals plants is considered here in details.

There will be a base model scenario from which other scenarios will emerge from the assumption that market demand are assumed as 10%, 20% higher demand, and 10%, 20% lower than the one base scenario in the subsequent time periods.

Each scenario can be defined using superscript $S = 1,2,3,4$ *and* 5 representing:

1 = 10% low base 2 = 20% low base 3 = base
4 = 10% high base 5 = 20% high base

This assumption means that the five scenarios have equal probabilities of $\frac{1}{5}$, hence ($E_\xi = \left\{\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right\}$).

F. The objective function

The objective function of the stochastic optimisation model can be presented by modifying the equation (1) to account the uncertainty.

$$Z\ \min\Big\{\{[\text{Production cost of crude oil}] + [\text{Production cost of final product}]\}$$
$$+ \frac{1}{5}\Big\{[\text{Transportation cost of crude oil}]$$
$$+ [\text{Stoarge cost}]$$
$$+ [\text{Transportation cost of final product}] + [\text{Penalty of shortage products}]$$
$$+ \begin{bmatrix}\text{Backlog penalty} \\ \text{of product}\end{bmatrix} - [\text{Sale revenue}]\Big\}\Big\}$$

$$Z = \min\left\{\left|\sum_{i\in I}\sum_{t\in T} CO_i \cdot Q_{i,t}\right| + \left|\sum_{j\in J}\sum_{t\in T} C_j \cdot V_{j,t}\right| + \right.$$
$$\frac{1}{5}\sum_{s=1}^{5}\sum_{t\in T}\left[\sum_{i\in I} TC_i \cdot TV_{i,t}^{s}\right] + \left|\sum_{j\in J} SC_j \cdot SV_{j,t}^{s}\right| + \left|\sum_{j\in J}\sum_{md\in MD} CT_j \cdot F_{j,md,t}^{s}\right| +$$
$$\left.\left|\sum_{j\in J}\sum_{md\in MD} \beta_j \cdot DS_{jd,t}^{s}\right| + \left|\sum_{j\in J}\sum_{md\in MD} CB_{j,md} \cdot VB_{j,md,t}^{s}\right| - \left|\sum_{j\in J}\sum_{md\in MD} PS_j \cdot F_{j,md,t}^{s}\right|\right\} \tag{14}$$

G. Constraints

The constraints used for the deterministic model are modified to the stochastic model formulation for each scenario $s \in \{1,2,3,4,5\}$.

To introduce uncertainty in market demand for final products, the demand balance equation (4) to (6) become:

$$\sum F_{j,md,t}^{s} \leq D_{j,md,t}^{s} \qquad \forall_j \in J, md \in MD,\ t \in T$$
$$s \in \{1,2,3,4,5\} \tag{15}$$

For the below average with 10%, 20% in product demand scenario $s = 1,2$:

$$VB_{j,t-1}^{s} + (1-\alpha)D_{j,md,t}^{s} = \sum F_{j,md,t}^{s} + DS_{j,md,t}^{s} +$$
$$B_{j,md,t}^{s}$$
$$\forall_j \in J, md \in MD,\ t \in T, s \in \{1,2,3,4,5\} \tag{16}$$

$$DS_{j,md,t}^{s} = \delta \left(VB_{j,t-1}^{s} + (1-\alpha)D_{j,md,t}^{s}\ \sum F_{j,md,t}^{s} \right)$$
$$\forall_j \in J, md \in MD,\ t \in T, s \in \{1,2,3,4,5\} \tag{17}$$

Where α is the degree of uncertainty. For each scenario assumed in the final products demand $\alpha =$ 0.1, 0.2

For the average in product demand scenario $s = 3$:

$$B^s_{j,t-1} + D^s_{j,md,t} = \sum F^s_{j,md,t} + DS^s_{j,md,t} + B^s_{j,md,t}$$
$$\forall_j \in J,\ md \in MD,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (18)$$

$$DS^s_{j,md,t} = \delta\left(VB^s_{j,t-1} + D^s_{j,md,t} - \sum F^s_{j,md,t}\right) \geq 0$$
$$\forall_j \in J,\ md \in MD,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (19)$$

For the above average with 10%, 20% in the product demand scenario $s = 4{,}5$:

$$B^s_{j,t-1} + (1+\alpha)D^s_{j,md,t} = \sum F^s_{j,md,t} + DS^s_{j,md,t} + B^s_{j,md,t}$$
$$\forall_j \in J,\ md \in MD,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (20)$$

$$DS^s_{j,md,t} = \delta\left(VB^s_{j,t-1} + (1+\alpha)D^s_{j,md,t} - \sum F^s_{j,md,t}\right) \geq 0$$
$$\forall_j \in J,\ md \in MD,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (21)$$

$$\sum VB^s_{j,md,t} \leq \sum(\lambda V_{j,t})$$
$$\forall_j \in J,\ md \in MD,\ t \in T \quad (22)$$

$$\sum DS^s_{j,md,t} \leq \sum(\delta D^s_{j,md,t})$$
$$\forall_j \in J,\ md \in MD,\ t \in T \quad (23)$$

The stochastic formulation of storage constraints:

$$\sum SV^s_{j,t} \leq \sum SV^{max}_{j,t} \qquad \forall_j \in J,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (24)$$

The stochastic formulation of transportation constraints:

$$TV^s_{i,t} \leq Q_{i,t} \qquad \forall_i \in I,\ t \in T,\ s \in \{1,2,3,4,5\} \quad (25)$$

$$F^s_{i,md,t} \leq TP^{max}_{j,t}$$
$$\forall_j \in J,\ md \in MD,\ t \in T \quad (26)$$

The stochastic formulation of yield products becomes:

$$\sum V^s_{j,t} = \sum(\gamma Q^s_{j,t}) \qquad \forall_j \in J,\ t \in T \quad (27)$$

IV. RESULTS AND DISCUSSION

To illustrate the key performance of the designed optimisation models, a number of case studies were carried out. “Table 1” lists the case studies selected for analysis and discussion.

(Case 0) represents the solution of the deterministic model before considering the effect of uncertainty of the market demand on the proposed supply chain. The rest of the cases (case 1 to case 8) explain different scenarios considered changes in the key chosen parameters. The changes in the optimal profitability of case studies compared with the optimal profitability of deterministic model (case 0) are showed in Table 1.

Table 1. Changes in Optimal Profitability of Case Studies Compared withcase-0

Case studies	Description	Change %
Case 0	Deterministic, base case	0.0
Case 1	Deterministic, - 20% market demand	-4.5
Case 2	Deterministic, - 10% market demand	7.7
Case 3	Deterministic, + 10% market demand	13.5
Case 4	Deterministic, + 20% market demand	-7.9
Case 5	Stochastic, - 20% market demand	8.4
Case 6	Stochastic, - 10% market demand	21.9
Case 7	Stochastic, + 10% market demand	21.9
Case 8	Stochastic, + 20% market demand	8.4

A. Deterministic base case (Case 0)

The deterministic base case results represent the considered to be optimal supply chain plan for which all parameters are considered at certain condition. The main points of Case 0 results are summarised in the following:

- The quantity of crude oil production and quantity of crude oil transported have the highest contribution in the overall quantities of the petroleum supply chain, which recorded 26.90% alike. It is followed by volume of refinery productions and volume of refinery products shipped with 25.58% and 16.96% respectively. While the lowest contribution quantities are represented by volume of backlog, volume of stored products and shortage product (below demand) with 1.79%, 1.72% and 0.09%, respectively. The contribution of each parameters of the supply chain is illustrated in” Figure. 3”.

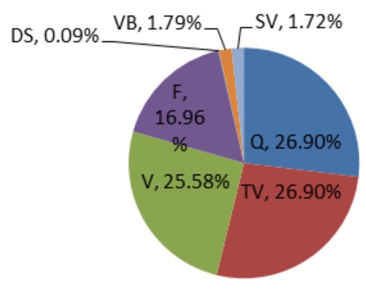

Figure 3. Optimal Quantities of Supply Chain Parameters (Case 0)

- The optimal quantity of the crude oil production is accomplished for all time periods during the planning horizon as shown in Table 2. The quantity gained from running the deterministic model Case 0 is 2.57E+07 tonnes of crude oil during time period of planning horizon which equivalent to 510,000 barrels/day. This quantity will be used in the simulated model proposed in this research, which will be explained in the next section for calculating other key performance measures of petroleum supply chain.

Table 2. Optimal Quantities of Supply Chain Parameters During Planning Horizon (tonnes)

Items	Optimal Quantities (tonnes)
Quantity of crude oil (Q)	2.57E+07
Quantity of transported crude oil (TV)	2.57E+07
Quantity of products (V)	2.44E+07
Quantity of shipped products (F)	1.62E+07
Quantity of shortage demand (DS)	90000
Quantity of backlog (VB)	1.71E+06
Quantity of product kept in stock (SV)	1.65E+06

- The contribution of each cost item to the overall cost of the supply chain is shown in “Figure. 4”. It is obviously that the cost of production quantity is the highest cost of the overall supply chain and represents more than half of the total costs of supply chain items followed by crude oil production cost with about one third of the overall costs of items.

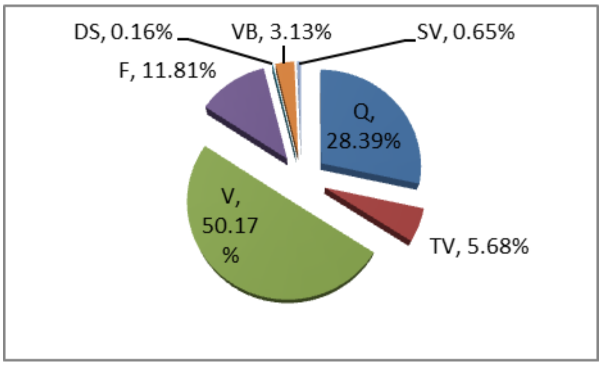

Figure 4. Contribution of Each Cost Items of the Supply Chain

- The average of the refinery products shipped in contrast with market demand is shown in ‘Figure. 5”. For example, lack of kerosene was almost 8%, LPG, Gasoline, and diesel were 10% each while 11.4% was the reported lack for the heavy fuel oil. The reason for this is due to presence of backlog and shortage demand quantities.

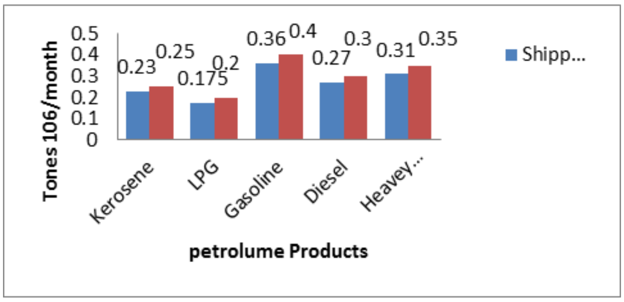

Figure 5. Average shipments of Refinery Products and Corresponding Market Demand (Case 0) Tonnes/Month

V. SENSITIVITY ANALYSIS

Uncertainty analysis and sensitivity analysis are essential parts of analyses for complex systems such as petroleum industry. Uncertainty analysis refers to the determination of the uncertainty in analysis results that derives from uncertainty in analysis of inputs, and sensitivity analysis refers to the determination of the contributions of each an individual uncertain analysis input to the uncertainty in analysis results [12].

Sensitivity analysis helps the decision maker by describing how changes in the state of nature probabilities and or/ changes in the payoff affect the recommended decision alternatives.

Two approaches were applied in studying the effect of uncertainty of the market demand on the supply chain. The first approach is based on introducing deviations in the deterministic model, and the scenario analysis stochastic approach is used for the second approach.

A measure tool known as Expected Value of Perfect Information (EVPI) is computed a maximum amount that a decision maker should pay for additional information that gives a perfect signal as to the state of nature. In the other words, EVPI represents the loss of a profit due to the presence of uncertainty or lack of information (AL Othman et.al. ,2008).

Mathematically, **EVPI** is calculated as the difference between the arithmetic average of optimum costs (value of objective function) of the five deterministic and stochastic plans (below 10%, 20%, average base and 10%, 20% above average).

The effect of uncertainty in market demand is studied through sets of cases studies and the results are showed in “Table 1”. The first set cases (Case1, Case2, Case3 and Case4) are solved by the deterministic model for ±10% and ±20% uncertainty in market demand.

Sensitivity analysis results indicate clearly that the optimum petroleum supply chain plans are sensitive to changes in market demand. Planning for a 20% decrease in market demand (Case1) is about 4.5 less profitability than the base case (Case 0), as well as the assuming 20% increase in market demand (Case 4) reduces the profitability by about 7.9% deviated on base case. In contrast, (Case 2 and Case 3) showed positive deviations in profitability about 7.7% and 13.54% respectively. Such a deviation in the profitability for each case study means that the planning of the supply chain under uncertain market demand is risky. Moreover, value of EVPI for both ±10% and ±20% deterministic plans (Case1, Case 2, Case 3 and Case 4) are more than 1.5% as seen in Figure 6.

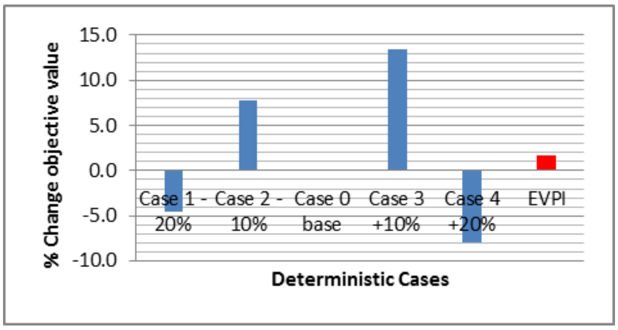

Figure 6 the Percentage Change of Objective Value Getting from Deterministic Cases.

Sensitivity analysis of the stochastic model cases (Case 5, Case 6, Case 7 and Case 8) shows that the stochastic optimisation model outputted rigid optimum supply chain plans with more deviations of profitability with to that of base case (Case 0). For a ±20% uncertainty in market demand (Case 5 and Case 8), has an increase in the profitability by 8.4% compared with base case. Whilst, the profitability has more positive deviation with 21.9% for ±10% uncertainty in market demand (Case 6 and Case 7) as shown in "Figure 7".

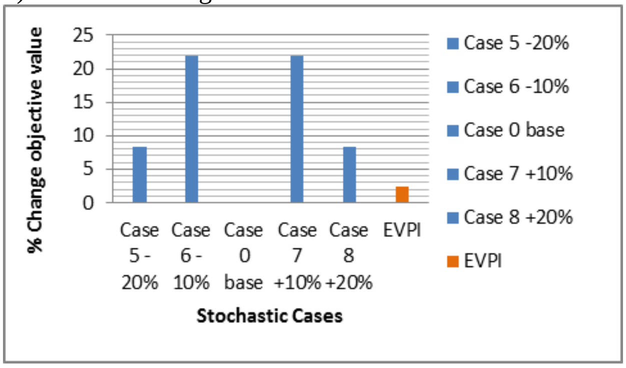

Figure7. Percentage Change of Objective Value Getting from Stochastic Cases.

EVPI for both ±10% and ±20% stochastic model plan calculated 2.4% of the base case objective value, which is higher than the value calculated for deterministic model plan. It means that the planning at stochastic conditions is more risky than the planning at the deterministic cases, although there is risky at both of them.

VI. CONCLUSION

The proposed network of a petroleum supply chain, which consists of all activities related to the petroleum industry from raw materials to distribution centres, has been designed and used as the basis of the proposed mathematical modelling purposes. A mathematical model of two-stage stochastic linear programming to address the strategic planning and optimization of petroleum supply chain has been developed and implemented to simulate and study the effect of uncertainty in the market demand for the valuable production on the proposed supply chain. Optimal planning results have illustrated the capabilities of the proposed mathematical model in developing a comprehensive one-year plans that ensure optimum operation of petroleum supply chain and maximum profitability. The key performance measurement considered in the mathematical model is that the cost of quantities of crude oil, transportation of crude oil, refinery production, production storage, production shipped, backlog and shortage demand.

Sensitivity analysis results have shown that planning in an uncertain of a market demand is risky. It is important for petroleum companies to develop and resilient supply chain plans in order to be able to capture the great benefit. Plans generated by the stochastic model have a significant high EVPI, which indicates that the plan of a supply chain by a stochastic program has more risk than the deterministic plan.

NOMENCLURE

Sets

I = Set of raw material (i)

J = Set of products (j)

MD = Set of market demand (md)

T = set of time period in the planning horizon for one year (t)

S = set of scenarios (s)

Variables

$Q_{i,t}$ = Volume of crude oil produced during period time(t).

$V_{i,t}$ = Production volume of product (j) at the end of period time (t).

$TV_{j,t}^{s}$ = Volume of crude oil (i) transported at the end of time period (t) under scenario (s).

$SV_{j,t}^{s}$ = Volume of product (j) kept in stock at the end of period time (t) under scenario (s).

$F_{j,md,t}^{s}$ = Volume of product (j) shipped to source demand (md) at the end of period time (t) under scenario (s).

$VB_{j,d,t}^{s}$ = Backlog quantities of product (j) for demand source (md) at the end of period time (t) under scenario (s).

$DS_{j,md,t}^{s}$= Shortage amount of product (j) for demand source (md) at the end of period time (t) under scenario (s).

$D_{j,md,t}^{s}$ = Demand quantity of product (j) for demand source (md) at the end of period time (t) under scenario (s).

$Cp_{i,t}$ = Maximum capacity of crude oil production.

$SV_{j,t}^{max}$ = Maximum allowed stock volume of product (j) at the end of time period (t).

$TP_{j,t}^{max}$ = Maximum capacity of transportation of products (j) shipped to market demand (md) at the end of period time (t).

Parameters

C_j = unit production cost for product (j) .

PS_j = price of product(j).

CB_j = Unit penalty of product (j).

CO_i = unit production cost of crude oil (i) .

CS_i = unit storage cost of product (j).
TC_i = transportation cost of crude oil(i).
CT_j = transportation cost of product (j).
$\beta_{j,d}$= penalty of shortage below demand of product (j).

Scalar
δ = is expected shortage demand fraction of unserved cumulative demand at any time period for a given demand source.
γ = yield of product.
λ = maximum backlog allowed.

REFERENCES

[1] Lababidi, H. M., Ahmed, M. A., Alatiqi, I. M., & Al-Enzi, A. F. (2004). Optimizing the supply chain of a petrochemical company under uncertain operating and economic conditions. *Industrial & Engineering Chemistry Research,43*(1), 63-73.
[2] Das, S. K., & Abdel-Malek, L. (2003). Modelling the flexibility of order quantities and lead-times in supply chains. International Journal of Production Economics, 85(2), 171-181.
[3] Al-Othman, W. B., Lababidi, H. M., Alatiqi, I. M., & Al-Shayji, K. (2008). Supply chain optimization of petroleum organization under uncertainty in market demands and prices. *European Journal of Operational Research, 189*(3), 822-840.
[4] Grossmann, I. E., Van Den Heever, Susara A, & Harjunkoski, I. (2002). Discrete optimization methods and their role in the integration of planning and scheduling. AIChE Symposium Series, 150-168.
[5] McDonald, C. M., & Karimi, I. A. (1997). Planning and scheduling of parallel semicontinuous processes. 1. Production planning. Industrial & Engineering Chemistry Research, 36(7), 2691-2700.
[6] Perea, E., Grossmann, I., Ydstie, E., & Tahmassebi, T. (2000). Dynamic modeling and classical control theory for supply chain management. Computers & Chemical Engineering, 24(2), 1143-1149
[7] Lyer, R. R., & Grossmann, I. E. (1998). A bi-level decomposition algorithm for long-range planning of process networks. Industrial & Engineering Chemistry Research, 37(2), 474-481.
[8] Moro, L., Zanin, A., & Pinto, J. (1998). A planning model for refinery diesel production. Computers & Chemical Engineering, 22, S1039-S1042.
[9] Dantzig, G. B. (1955). Linear programming under uncertainty. *Management Science*, *1*(3-4), 197-206.
[10] Vladimirou, H., & Zenios, S. A. (1997). Stochastic linear programs with restricted recourse. *European Journal of Operational Research, 101*(1), 177-192.
[11] Sahinidis, N., Grossmann, I., Fornari, R., & Chathrathi, M. (1989). Optimization model for long range planning in the chemical industry. Computers & Chemical Engineering, 13(9), 1049-1063.
[12] Helton, J. C., Johnson, J. D., Sallaberry, C. J., & Storlie, C. B. (2006). Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliability Engineering & System Safety, 91(10), 1175-1209.

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