



# Simple Linear Regression (SLR) Model for Re-Bending Behavior of a Non-Crimp Dry Thick Carbon Fiber Fabrics

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**Abstract**— During manufacturing process, fabric such as the non-crimp dry thick fabric (NCF) is bent and re-bent many times until the fabric takes the desirable shape in the mold. Understanding the behavior of dry NCFs requires conducting experimental bending tests which is expensive due to the cost of the carbon fabrics. This paper aimed to model the bending behavior of NCF using simple Linear Regression Model in which the bending moment force is used as a response to displacement. The data modeled were obtained from testing three samples each sample was tested three consecutive times. A total of nine simple linear regression models were created. These nine models showed strong correlation between bending force and extension. The results also showed that the fabric becomes more flexible when it is subjected to re-bending process. Good performance of these models was confirmed using cross-validation method indicating that all presented models in this study were able to predict the bending behavior of non-crimp dry thick fabric.

**Index Terms:** Bending behavior, dry thick carbon fibers, non-crimp fabrics, Simple Linear Regression model.

## I. INTRODUCTION

Non-crimp fabrics (NCFs) have the potential to be used in manufacturing next generations of composite materials in production of aerospace parts [1]. Non-crimp fabrics are manufactured by laying one or more layers of parallel crimp-free fibers that are assembled by means of stitching, knitting, or bonding [2].

The bending behavior of thick fabrics is one of the most important properties that used to define the quality of the final product. During the manufacturing process of aerospace composites parts, bending the carbon fabric into the mold is needed to take the shape desired of the final part. The ability of the thick fabric to bend and re-bend without wrinkling or damaging will define the flexibility of fabric during the manufacturing process.

To understand the behavior of dry NCFs, several experimental bending tests are conducted for different samples of dry NCFs [3]. The experimental data were analyzed using a well-known statistical method called Taguchi method [4,5]. The experimental tests are, however, expensive to conduct due to the cost of the carbon fabrics. Therefore, creating a model to predict the behavior of such non-crimp dry thick fabric under bending load using simple linear regression (SLR) method is proposed in this research. In this study, the behavior of dry NCFs is characterized by re-bending the same sample three consecutive times. The SLR method has an advantage of being one of the most reasonable and simple method to study the relationship between two variables. It provides useful results even with relatively fewer data, and has less laborious, therefore cost-effective [6,7].

## II. METHOD

The effect of re-bending a non-crimp dry thick carbon fiber fabric on the bending force was investigated in this paper using SLR model. Three 2mm-thickness samples were constructed by laying carbon fiber yarns in layers that are running in bidirectional and symmetrical manner. The constructed bidirectional layers were bonded together by stitching as shown in Figure 1. The manufacturing process of such non-crimp dry thick fabrics were explained in detail in previous work [3].

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Figure. 1: Non-crimp dry thick fabrics samples.

### 2.1 Test setup

Samples were treated as a cantilever during the bending test as shown in Figure 2. During the test, the bending force  $F_B$  was applied by moving the crosshead of the machine downward (i.e., in the direction of displacement). The movement of the crosshead was represented as a displacement and the corresponding amount of bending force was recorded. Therefore, the amount bending force was the main response recorded corresponding to the displacement. For more details on the experimental work see reference [3].

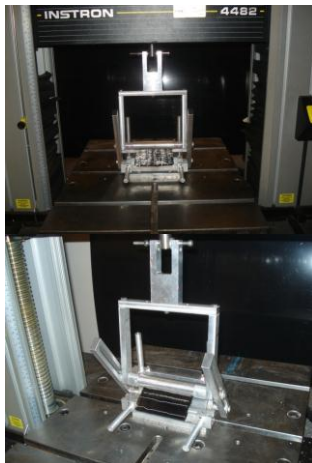


Figure.2: The bending rig attached to the testing machine.

The value of bending force ( $F_B$ ) resulting from testing the first sample (S1) for the first time (T1) was recorded and labeled as  $F_B$  for (S1-T1). When the first sample was tested for the second time, it was labeled as (S1-T2), and when it tested for the third time, it was labeled as (S1-T3). The same procedure was conducted on the second sample (S2) and third sample (S3). As a results, a total of nine tests were conducted based on the three samples. Figure 3 shows the flow-chart of the bending test

procedure conducted on the three samples of 2mm thickness.

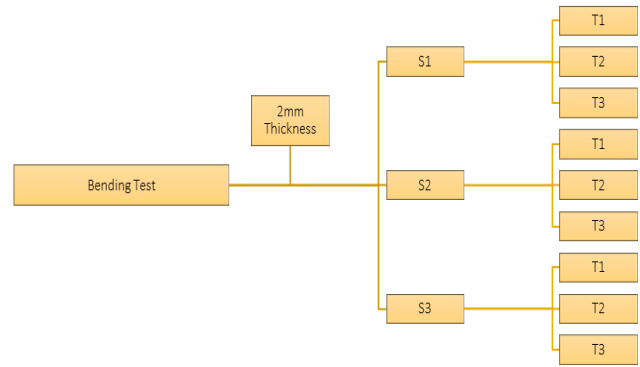


Figure. 3: Flow-chart of the bending test procedure.

### 2.2 The linear regression model:

Linear Regression is a statistical method that predict one dependent variable using one or more independent variables. If the model has one independent variable, it is called simple linear model (SLR). While it is called multiple linear regression, when it has many independent variables. The first order regression model has the following formula [6]:

$$y = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \epsilon$$

where, for  $i = n$  observations:

$y_i$  is the dependent variable,  $x_i$  is the independent variable,  $\beta_0$  is the y-intercept,  $\beta_p$  is the slope coefficients, and  $\epsilon$  is the model's error term (residuals).

Since this study investigates the relationship between one dependent variable (bending force) and one independent variable (displacement), a simple linear regression formula will be used as presented below,

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

where,  $y_1$  is the dependent variable (bending force),  $x_2$  is the independent variable (displacement), and  $\epsilon$  is the model's error term (residuals)

### 2.3 Data exploration and SLR assumptions:

Boxplots, which are used in this study, are a common way to explore the data before conducting the SLR to view the differences between the data sets. The major assumptions that are associated with a linear regression model are: the linear relationship, the normality distribution, and the homoscedasticity [8].

The assumption of linear relationship is often examined using scatter plot to identify correlation between the dependent variable (y), represented here in this work by the bending force value, against the independent variable (x), which represented here by the displacement. Linearity can also be tested using residuals versus fitted-values plot [9]. Normality means that the observations are normally distributed. This can be checked using normal Q-Q plot represented in section 3.3. Homoscedasticity which means that the error from predicted values will not change significantly over the

range of the model's prediction. Homoscedasticity is to test for the equality (homogeneity) of variance across groups, using tests such as F-test, Bartlett's test, Levene test, or Fligner-Killeen test [10-12]. In this study, both linearity and homoscedasticity will be evaluated using residuals versus fitted-values plot. Note that linearity, normality, and homoscedasticity are to be tested after conducting the models.

The SLR model was used in this study to investigate the association between bending force and the displacement with corresponding confidence level of 95%. The R-squared was used to show how well the model explains the observed data.

To further test the model performance, cross-validation method has been conducted for each data sample by splitting the dataset into 2 parts- training data and testing data. Training data is used to generate a model, while testing or validation data is used to evaluate the model's performance. Ultimately, a good model would present high accuracy on the testing data (as unseen data). In this study the model is conducted using random selection of 70% of the data. Then the bending force were predicted using the remaining 30% of the data. In this work, three models (T1-Model, T2-Model, and T3-Model) were created from each bending test of the three samples, therefore the cross-validation method was conducted on each model. For example, for the first test model (T1-Model) a cross-validation was conducted on the represented model data and R-squared was reported. The performance of the model is then evaluated using three statistical metrics: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Correlation between predictions and observations.

One of the advantages of the cross-validation method is that both training and testing data well represent the original data due to the random selection. The main disadvantage of this method is that the algorithm at each repetition must train the model from the beginning and therefore increasing the computation time. However, this will not be a problem in our study due to the reasonable data size.

### III. RESULTS

#### 3.1 Data exploration

Before investigating the relationship between bending force and the extension, data are being explored to have a closer look at the dependent variable which is the bending force. As mentioned before, the data in this study consist of three samples and each sample was subjected to three tests. In this section, three sets of data have been explored, namely T1, T2 and T3 to see whether the test number would have any effect on the relationship between bending force and the extension. Figure 4 shows a comparison between bending force values of test one (T1) for the three samples. It can be seen that S2 and S3 have higher bending force readings than S1. Figure 5 shows the boxplots of the bending force readings for the same three samples but during T2. There is a small difference between S1 and S2 bending force values, but S3 shows less bending force values. During the third test T3 shown in Figure 6, bending force

readings of the three samples are quite similar to those of T1.

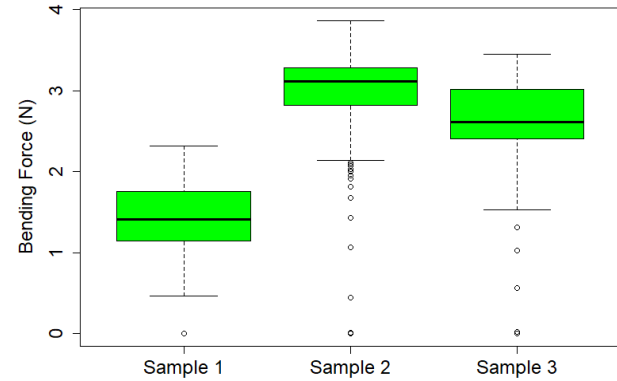


Figure. 4: The bending force of the three samples during T1.

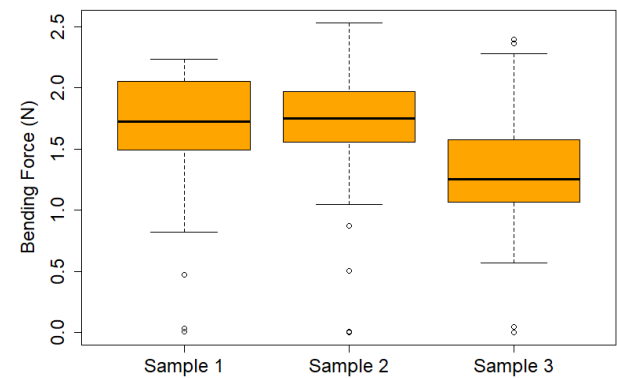


Figure. 5: The bending force of the three samples during T2.

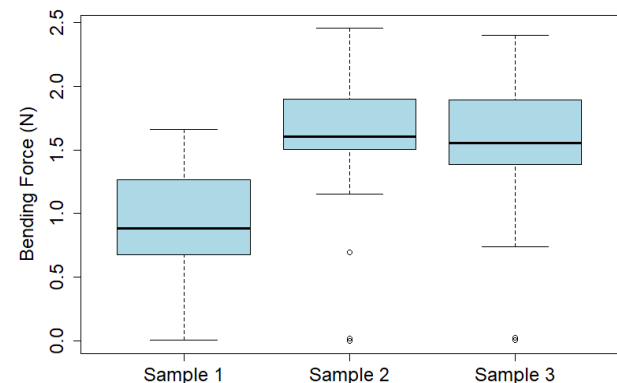


Figure. 6: The bending force of the three samples during T3.

#### 3.2 Simple Linear Regression models (SLR)

The behavior of bending force has been modeled using SLR method. SLR for each test and the results are presented in this section.

##### 3.2.1. First Test (T1)

For the first test (T1), the SLR models for all three samples (S1, S2, and S3) are shown in Table 1. It can be seen that the p-value less than 0.05 and a R-squared values ranged between 0.67 and 0.80. The first sample (S1) showed a low value of the Y-intercept (2.05) comparing with the other two samples S2 (3.62) and S3 (3.62). This difference is due to the manufacturing process of the samples. More specifically, S1 was manufactured manually by laying fibers by hand while S2 and S3 were manufactured using automated technique.

Therefore, the quality of S2 and S3 was better than S1. This is in a good agreement with the boxplots in Figure 4 in which showed low bending force values for S1 comparing with S2 and S3. As a result, the S1 model is ignored in this study. The T1 model for all samples can be represented by averaging only S1 and S2 models.

Table 1: SRL models of the first test T1 for all three samples

Test 1	Models	P-value	R-squared
Sample 1	$Y = 2.05 - 0.038 x_1$	<0.05	0.80
Sample 2	$Y = 3.62 - 0.033 x_1$	<0.05	0.67
Sample 3	$Y = 3.62 - 0.033 x_1$	<0.05	0.77
T1-Model (average)	$Y = 3.62 - 0.033 x_1$		

### 3.2.2. Second Test (T2)

SRL models for all three samples (S1, S2, and S3) in T2 are shown in Table 2. The three SRL models also showed significant p-value less than 0.05 but S1 model showed R-squared value of 0.34, therefore, the first sample (S1) is not considered in this study. This deference is also due to a defect of the manufacturing process. As a result, the representative SLR model of the second test T2 is obtained based on the average of both S2 and S3 models.

Table 2: SLR models of the second test T2 for all three samples

Test 2	Models	P-value	R-squared
Sample 1	$Y = 2.05 - 0.02 x_1$	<0.05	0.34
Sample 2	$Y = 2.22 - 0.029 x_1$	<0.05	0.61
Sample 3	$Y = 1.77 - 0.0285 x_1$	<0.05	0.77
T2-Model (average)	$Y = 2.0 - 0.029 x_1$		

### 3.2.3. Third Test (T3)

Similarly, SLR models of the third test (T3) for all three samples (S1, S2, and S3) are shown in Table 3. The SLR model for the third test (T3) is, however, estimated by averaging all three sample SLR models because they showed significant p-values less than 0.05 and R-squared values ranging from 0.56 to 0.70.

Table 3: SLR models of the third test T3 for all three samples

Test 3	Models	P-value	R-squared
Sample 1	$Y = 1.44 - 0.032 x_1$	<0.05	0.70
Sample 2	$Y = 2.084 - 0.025 x_1$	<0.05	0.56
Sample 3	$Y = 2.07 - 0.029 x_1$	<0.05	0.63
T3-Model (average)	$Y = 1.86 - 0.029 x_1$		

Based on p-values, all nine SLR models showed strong correlation between bending force and extension. The R-square values also indicated that the two variables are correlated except for the first sample of the second test (S1-T2).

### 3.3 Cross-validation Method

For Test 1 (T1) dataset, the SLR model was generated based on randomly selected 70% of the data with significant R-squared of 0.79. Then this model was used to predict bending forces using the 30% remaining data. The correlation between predictions and measured data was 0.82 indicating a good-agreement and well model performance. This good agreement is further confirmed by the small values of the RMSE (0.16) and the MAE (0.11). These two values mean, on average the square root of the variance of the residuals is 0.16, and the forecast's distance from the true value of bending force is 0.11. respectively.

Similarly, for Test 2 – (T2) data, the 70% based model, (R-squared = 0.69) was able to predict the bending force based on the remaining 30% data. A correlation between predictions and observations in this case was 0.73 which confirms the model's strong performance, with RMSE of 0.15 and MAE of 0.11. For T3, the 70% training data generates the model (R-squared is 0.71). When it was tested on the remaining data, the correlation between predictions and observations was 0.68 indicating again well-agreement and well model performance supported by RMSE of 0.18 and MAE of 0.14.

After building a linear regression model for each test, it is important to examine whether the models meet the assumptions of linear regression. Figure 7 shows the linear relationship examination of the SLR model for three tests. Figure 7a, b, c represents the residuals versus fitted values for test (T1), test (T2), and test (T3) respectively. It can be seen that the residuals bounce randomly around the 0-line suggesting that the assumption of linearity is reasonable. The normal Q-Q plot as shown in Figure 8a, b, c is to test the normality assumption for test (T1), test (T2), and test (T3) respectively. The data are close to a straight line indicating that the assumption of normality is valid. Finally, the residuals show no bias, explained by the roughly horizontal band of residuals formed around the 0 line. This suggests that the variances of the error terms are equal. Therefore, all SLR models fit the assumption of homoscedasticity (Figure 7a, b, c).



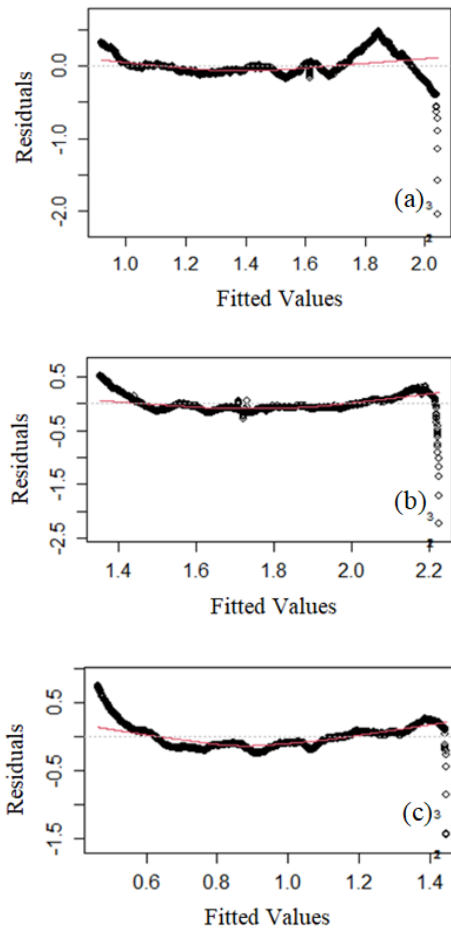


Figure 7: Linearity assumptions of the SLR models for: (a) test T1, (b) test T2, and (c) test T3.

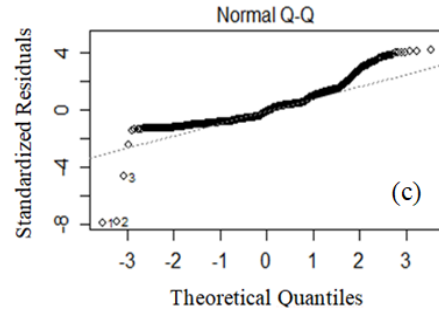
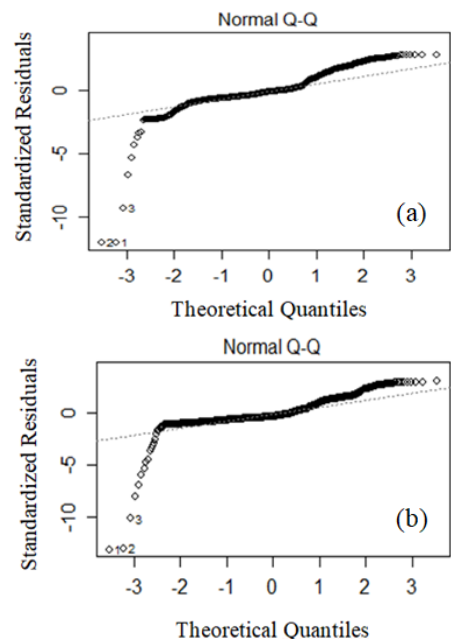


Figure 8: Normality assumptions of the SLR models for: (a) test T1, (b) test T2, and (c) test T3.

#### IV. DISCUSSION AND CONCLUSIONS

During manufacturing process, the fabric such as the non-crimp dry thick fabric is bent and re-bent many times until the fabric takes the desirable shape in the mold. Using LSR models to study the effect of re-bending of samples emulates the process of laying the fabric into the mold during manufacturing of composite parts.

Figure 9 shows the three representative SLR test models. Each SLR test model was generated from averaging the reliable sample models. T1-Model, T2-Model, and T3-Model represent the bending behavior of the same fabric when bent for the first time, second time, and third time, respectively.

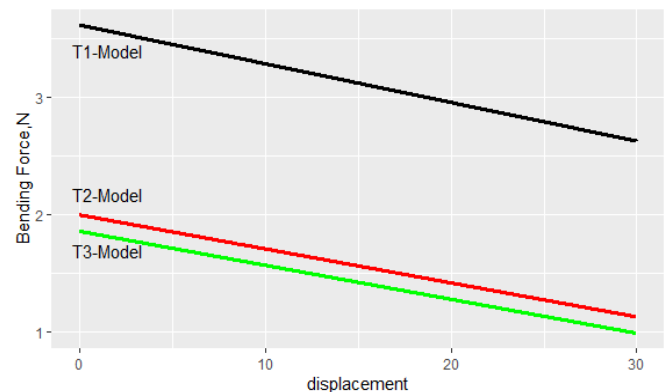


FIGURE 9: A comparison between the SLR test models: T1, T2, and T3.

One can see clearly, each model shows the bending behavior of a non-crimp thick fabric has a linear relationship with negative slope. This verifies the fact that the force needed to bend thick fabric at the ending edge is less than the beginning edge. It shows also that the y-intercept value (i.e., the slope of each test model) decreases at each time the fabric being bend. This decreasing in y-intercept values is related to the fact that during each bending process a compaction and a rearrangement of yarns are occurred. Therefore, the fabric becomes more flexible when it is subjected to re-bending process.

Each model was verified in two ways. The first way of verification was by comparing models that were created from three different samples at the same test setup. The resulted models were consistent. The second way of verification was by successfully passing the cross-validation condition used in such simple regression

method to create a reliable model. In conclusion, the resulted models were able to predict the bending behavior of such non-crimp dry thick fabric.

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