Finite Element Analysis based Stress Intensity Factor Solutions for Surface Cracks

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Abstract— **The stress intensity concept is important in terms of crack extension as critical values of the stress intensity factors govern crack initiation. Therefore, the present work determines stress intensity factors for semielliptical shallow and deep surface cracks as a function of parametric angle, crack depth, and aspect ratio for tension and bending loads. The stress intensity factors are obtained from a three-dimensional finite-element analysis of semielliptical surface cracks in finite plates subjected independently to tension and bending loads under elastic conditions. The obtained stress intensity factor is used to predict the crack growth under linear elastic conditions. Results show that the stress intensity factor varies along the crack front for shallow and deep cracks. At the deepest point in bending and tension the stress intensity factor increases as the ratio of the crack depth to the crack length at surface decreases. However, at the free surface the stress intensity factor becomes maximum when the crack depth equals crack length. A surface crack in tension loading is predicted to break the wall thickness with a relatively small amount of crack growth at surface. While in bending the crack breaks through with large amount of crack growth in the width direction.**

Index Terms: **finite element analysis, linear elastic fracture mechanics, surface cracks, stress intensity factor.**

I. INTRODUCTION

Fracture mechanics methodology determines the integrity of structures that contain cracks which if left will cause failure of the structure. One of fracture mechanics approaches is the Linear elastic fracture mechanics (LEFM) that characterises the crack tip stresses of a material under elastic conditions, typically material with elastic behaviour that fail under brittle fracture. The (LEFM) is one of the most successful developments of continuum mechanics and widely applied for analysis of structures containing cracks [1]. The relevant parameter used under (LEFM) is stress intensity factor (K) that characterises displacement, stress, and strain in the near field around a crack tip in a linear elastic solid [2]. Critical values of (K) govern the initiation of crack extension, and fatigue crack growth can also be analysed using variations in (K) [3].

Computation of (K) is therefore fundamental to any fracture or fatigue crack growth analysis. Most failures in practice are due to surface cracks where the Opening Mode mechanism of fracture (Mode I) is the favourite subject. The complex state of stresses exists near the crack tip, where the stresses undergo a sharp variation [4]. Consequently, these stresses can lead to unexpected failure. Due to the complexity of practical problems and the difficulty of the analysis, several integrity assessments of engineering structures remain using two-dimensional (2D) analysis. However flaws or cracks in engineering structures, especially in pressure vessels, piping and large hydraulic equipments, and structures are three-dimensional (3D) in nature. The (3D) solutions are necessary to provide more accurate evaluation on the strength of cracked structures and better understanding of the phenomenon of fracture [5]. Therefore, crack problems should be analyzed using (3D) fracture mechanics theories. From fracture analysis it is well known that the stress intensity factor (SIF) depends on crack length, crack front curvature, crack edge to free surface and structure configuration [6]. Furthermore, the evolution of the surface crack shape is important in engineering applications such as Leak-before-break. So, the crack shape predictions are usually studied by numerical models, which are based on determining (K) at different points along the crack front. In order to reduce the calculations, the authors in the literature prefer to assume that the crack front shape does not change during the fatigue growth process. Reference [7] studied the crack growth assuming that the crack front shape remains constant during the propagation. Researchers in [8–9] considered several points along the thickness, thus allowing variation of the crack shape during the process. They discussed the evaluation of (K) and its sensitivity to crack shape in circular and semi-elliptical crack using Finite Element (FE) models.

The present work aims to determine the stress intensity factor (K) for semi-circular and semi-elliptical surface cracks under Opening Mode. The crack shape predictions are also considered. The finite element analysis is used to characterise (K) in terms of load, geometry, and crack size.

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II. STRESS INTENSITY FACTOR

Consider a cylindrical coordinate system (r,θ) centred at the crack tip in an isotropic elastic material as illustrated in Figure. 1.

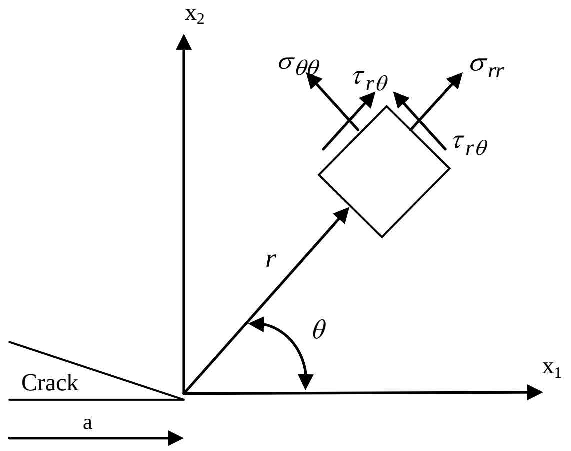

Figure 1. Stresses in the Polar Co-Ordinate System ahead of the Crack.

Westergaard [10] introduces an asymptotic solution for the stresses and displacements close to the crack tip. For Mode I loading, the leading stress term can be written as:

$$\sigma_{rr} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+\ldots\ldots\ldots$$

$$\sigma_{\theta\theta} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+\ldots\ldots\ldots \qquad (1)$$

$$\tau_{r\theta} = \frac{\sigma\sqrt{\pi a}}{2\pi r}\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)+\ldots\ldots\ldots$$

Where (*r*, *Θ*) are polar coordinates centred at the crack tip, and the radial distance *r* is very small compared to the crack length a.

Irwin [11] showed that the magnitude of stress at a crack tip can be characterized by a single parameter, the stress intensity factor, K:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \qquad (2)$$

This solution is asymptotic in nature with a singularity at the crack tip.

Assuming an isotropic material with linear elastic behavior, the stress field on any linear elastic body that holds a crack is described by stress intensity factor (K) that depends on type and level of loading, and the geometry. This parameter can be determined, according to [11], as follows:

$$K = \sigma\sqrt{\pi a}\ Y \qquad (3)$$

Where (Y) is the dimensionless factor of geometry and loading.

The stress intensity factor (SIF) is a physical quantity used as a control parameter to evaluate the critical state of a crack. So, accurate stress intensity factors for such cracks are necessary for reliable prediction of fatigue crack growth rates or fracture. The stress intensity factor solutions for semi-elliptic surface cracks are reported in [9], [12-15].

III. GEOMETRY AND MATERIAL

Deep and shallow semi-elliptical surface cracks in three points bending and tension have been studied. The geometry considered is shown in Figure. 2, and can be defined in a non-dimensional manner as:

(a/c=0.33, 0.5, 1, a/t=0.2, 0.5, W/a=8, H/a=16)

Where (*a*) is the crack depth, (*c*) the crack length at surface and the thickness is (*t*). The width and length of the model are (*2W*), (*2H*) respectively.

The material response is defined as isotropic elastic with Young’s modulus of (210 GPa), Poisson’s ratio of (0.3), and a yield strength of (300 MPa). Due to symmetry only one quarter of the geometry is modelled and symmetric boundary conditions are imposed on the appropriate surfaces.

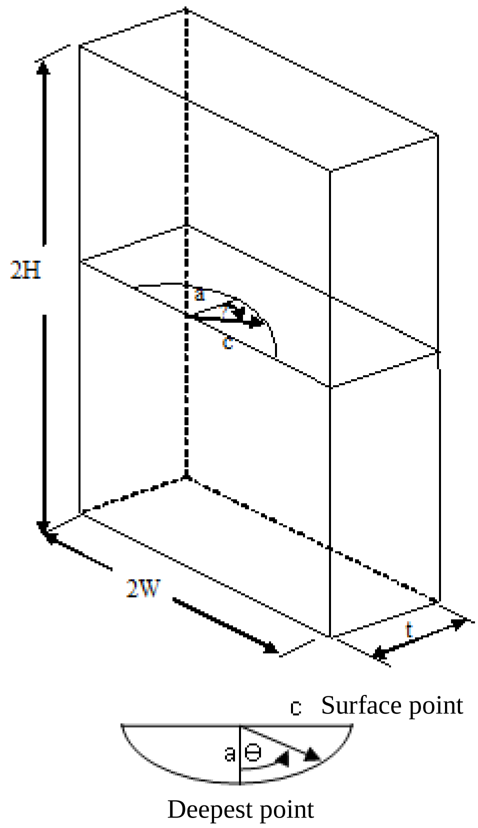

Figure 2. Illustration of the Surface Crack Geometry.

IV. FINITE ELEMENT MODEL

To capture the accurate stress profile near the crack a very refined mesh was used close to the crack front. Collapsed three dimensional linear continuum hexagonal elements with reduced integration were used with coincident but independent nodes. For deep crack model, thirty concentric rings of elements extended radially from the crack tip were created. Each ring contained (400) elements: (40) elements along the crack front and (10) around the half circumference. The average element size was approximately (*t/1000*) along the crack line, where *t* is the thickness. The elements were biased towards the free surface to accommodate stress gradients. The total number of elements was (107,672). For shallow crack model, thirty concentric rings of elements extended radially from the crack tip. Each ring contained (312) elements: (26) elements along the crack front and (12) around the half circumference. The average element size was approximately (*t/2000*) along the crack line. The total number of elements was (120,000). The finite element mesh is shown in Figures. 3, 4.

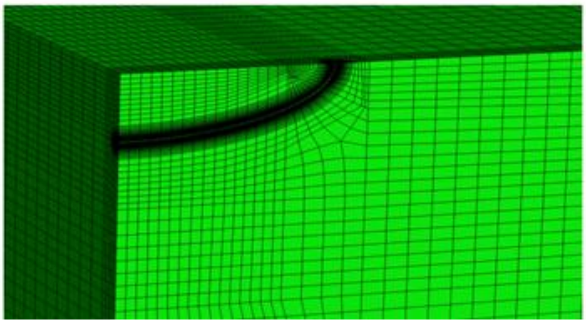

Figure 3. Finite Element (FE) Mesh of a Shallow Semi-Elliptical Surface Crack.

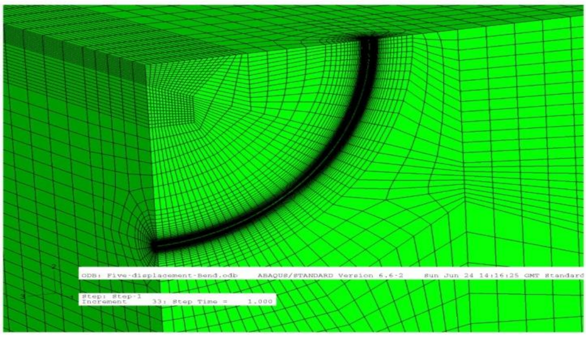

Figure 4. Finite Element (FE) Mesh of the Model of a Deep Surface Crack.

V. RESULTS

A. Finite Element Model Benchmarking

Three-dimensional finite element calculations of stress intensity factors for surface cracks with various aspect ratios are performed. The SIFs obtained were initially validated with those derived by the weight function method [15-17]. For convenience the stress intensity factor (SIK) is normalised by ($\sigma_{ap}\sqrt{\pi a/Q}$) as:

$$SIF = K/(\sigma_{ap}\sqrt{\pi a/Q}) \qquad (4)$$

where (σ_{ap}) is the applied stress, a is the crack depth, and (Q) the elliptical integral of the second kind and is calculated by the following approximate formula:

$$Q=1+4.593(a/c)^{1.65} \qquad (5)$$

The stress intensity factors calculated from the FEA model were benchmarked with the results by the weight function method (WFM). Figures. 5 and 6 show consistent results between two methods for shallow and deep semi-elliptical surface crack (a/t=0.2, 0.5) in bending and tension, respectively.

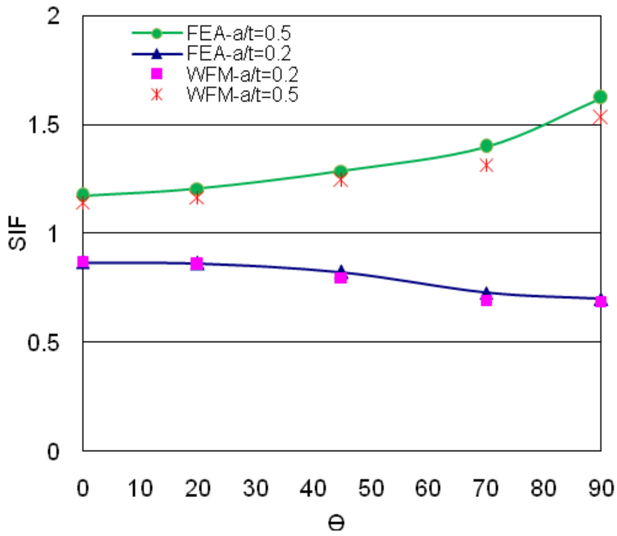

Figure 5. Benchmark of Stress Intensity Factor from FEA Model and Weight Function Method for Shallow and Deep Semi-Elliptical Surface Crack (a/t=0.2, 0.5, and a/c=0.33) Under Bending.

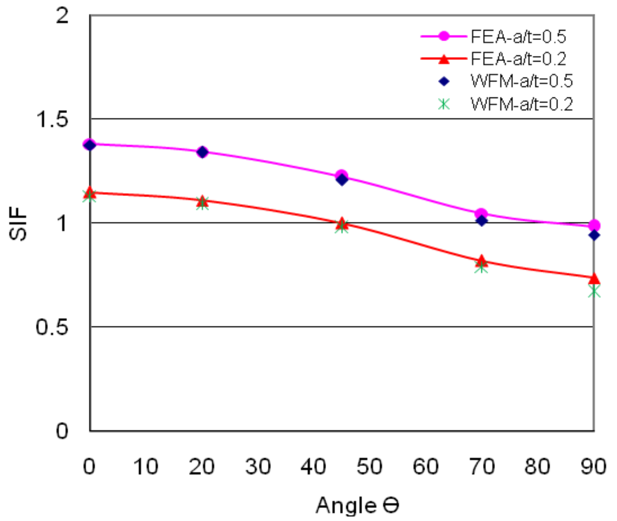

Figure 6. Benchmark of Stress Intensity Factor from FEA Model and Weight Function Method for Shallow and Deep Semi-Elliptical Surface Crack (a/t=0.2, 0.5, and a/c=0.33) Under Tension.

B. Stress Intensity Factor

Figure. 7 shows the normalised stress intensity factor (K) for shallow semi-elliptical surface cracks (a/t=0.2) with different aspect ratio (a/c= 0.33, 0.5 and 1) under bending. It can be seen that stress intensity factor is consistent around the crack front for aspect ratio of (0.5). However the variations in stress intensity factor are obvius for (a/c=0.33 and 1). For (a/c=0.33), large values occur at the deepest point in comparison to the free surface. Different profile are observed for (a/c=1) where the large stress intensity factor values occur at the free surface and reduced at the deepest point. Consequently, it is expected that the crack grows with greater rate at the deepest point than at the free surface as long as the crack length (c) is greater than the crack depth (a). In contrast, the surface crack can grow with larger amounts at the free surface when both the crack lemgth and crack depth are similar.

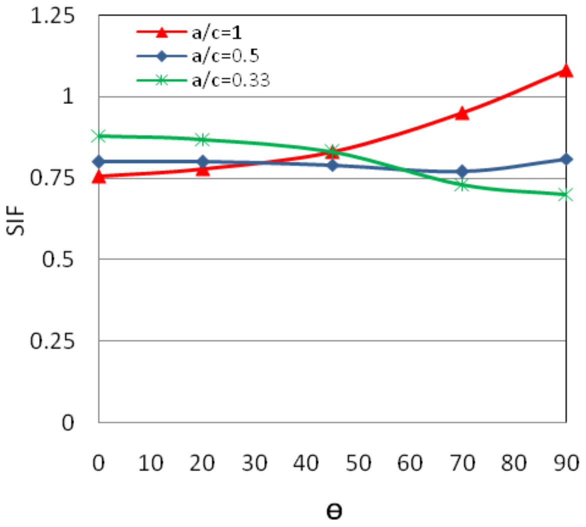

Figure 7. Stress Intensity Factor for Shallow Semi-Elliptical Surface Cracks (a/t=0.2, a/c=0.33, 0.5 and 1) Under Bending.

Figure. 8 shows the normalised stress intensity factor (K_I) as a function of the parametric angle (Ɵ) for deep semi-elliptical surface cracks (a/t=0.5) with different aspect ratio (a/c= 0.33, 0.5 and 1) under bending. Completely different profile is obtained for deep surface cracks in comparison to shallow ones. It is clear deep surface cracks develop larger SIFs associated with big variations along the crack front where large values occurred at the free surface and small values observed at the deepest point. This SIF profile under bending for elastic conditions can cause crack growth along the crack front with large amount at the free surface. This is due to the tensile field affects near the free surface which lead the crack opening while compressive bending filed will affect when the deepest point approached. The variation in SIF in deep cracks is different from shallow cracks since deep cracks showed maximum SIF at the free surface and the minimum at the deepest point, while in shallow cracks the maximum value was at the deepest point and the minimum at the free surface.

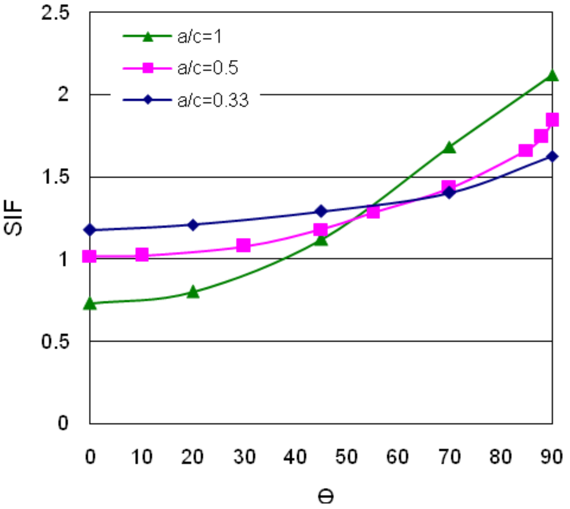

Figure 8. Stress Intensity Factor for Deep Semi-Elliptical Surface Cracks (a/t=0.5) Under Bending.

In tension loads, the stress intensity factor (SIF) for shallow semi-elliptical surface cracks (a/t=0.2) with aspect ratio (a/c= 0.33, 0.5 and 1) varies along the crack front as shown in Figure. 9. For (a/c= 0.33, 0.5) the stress intensity factor increases from the low values at the free surface to the deepest point when reaches its maximum. However, surface cracks with (a/c=1) shows less variation in SIF along the crack front with maximum SIF occurs at the free surface.

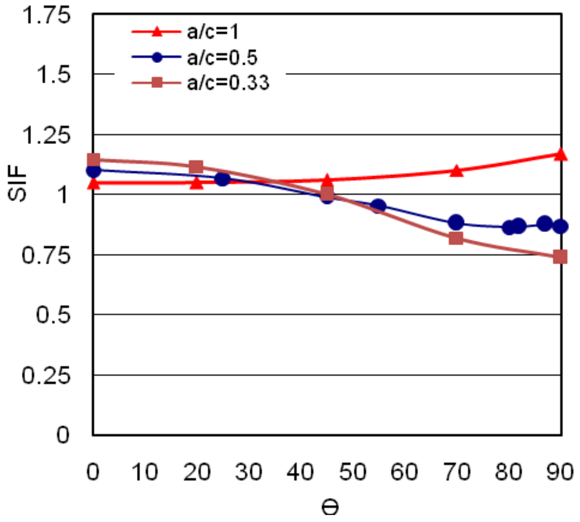

Figure 9. Stress Intensity Factor for Shallow Semi-Elliptical Surface Cracks (a/t=0.2, a/c=0.3, 0.5 and 1) in Tension.

Deep semielliptical surface cracks (a/t=0.5) under tension shows different profile in comparison to bending as shown in Figure. 10, where large SIFs occurred at the deepest point (Ɵ=0^o) and low values associated with the free surface for a/c≤0.5. Large stress intensity factor (SIF) developed for (a/c=0.33 and 0.5) in tension at the deepest point reduces as the free surface approaches and becomes smallest at the cross of the crack length with the crack front that is (Ɵ=90^o). However, this is no longer exist for (a/c=1) where the free surface developed higher SIF than the deepest point. It can also be seen that deep surface cracks develop higher SIF in contrast to shallow surface cracks. In addition, the increase in SIF at the deepest point is associated with the decrease in the aspect ratio (a/c) which means increasing the crack length with respect to the crack depth. So, for the crack growth in tension for materials with elastic behavior it is expected to extend largely at the deepest point ($\Theta=0^o$) for low aspect ratio (a/c=0.33 and 0.5), while the crack will extend large amount at the free surface for high aspect ratio (a/c=1). It clear that deep surface cracks (a/t=0.5) developed higher stress intensity factor along the crack front than shallow cracks (a/t=0.2), and the overall profile is much similar where the maximum SIF occurred at the deepest point while the minimum was at the free surface for (a/c≤0.5).

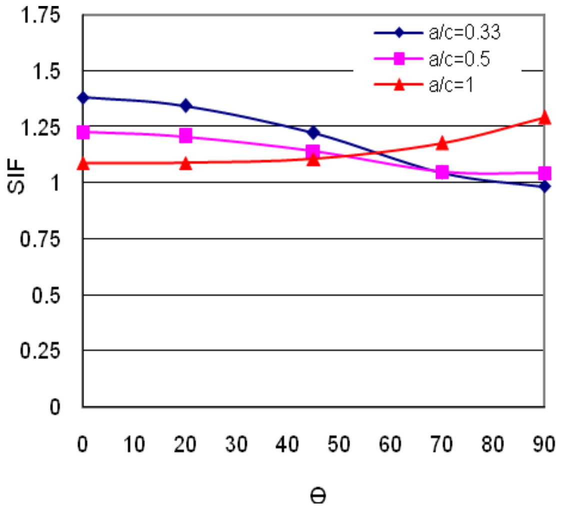

Figure 10. Stress Intensity Factor for Deep Semi-Elliptical Surface CRacks (a/t=0.5) in Tension.

The deepest point and the free surface point are particularly important for structural integrity assessments. Therefore, a comparison of the SIFs at the deepest point and the free surface of surface cracks in bending and tension is presented in Figures. (11) to (14). The SIFs for the deepest point under bending is shown in Figure. 11 for different crack geometries. It can be seen that deep surface cracks developed higher SIFs than shallow ones. Furthermore, higher stress intensity factor is associated with low aspect ratio (a/c<0.5). Different profile of stress intensity factor is developed at the free surface as shown in Figure. 13. The SIF increases for both of shallow and deep surface cracks as the aspect ratio (a/c) increases.

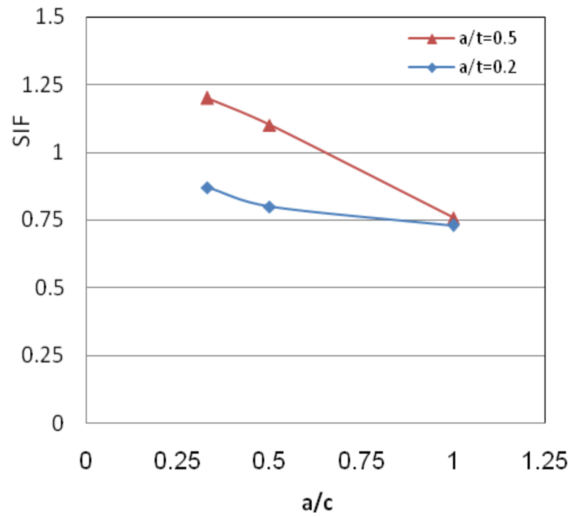

Figure 11. Stress Intensity Factor at the Deepest Point of Deep and Shallow Semi-Elliptical Surface Cracks (a/t=0.5, 0.2), as a Function of the Aspect Ratio a/c in Bending.

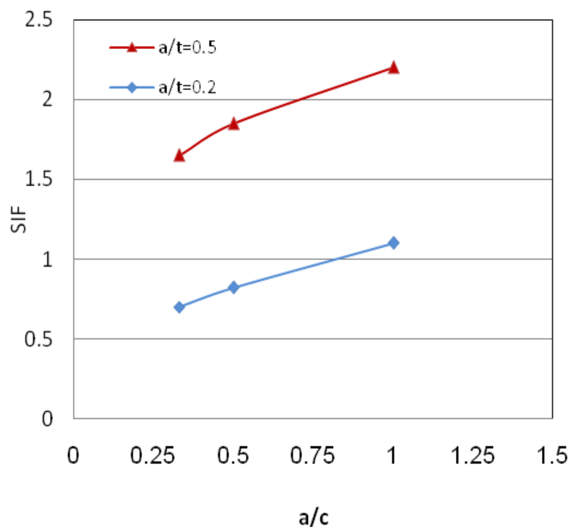

Figure 12. Stress Intensity Factor at the Free Surface of Deep and Shallow Semi-Elliptical Surface Cracks (a/t=0.5, 0.2), as a Function of the Aspect Ratio a/c in Bending.

In tension, the stress intensity factor at deepest point becomes higher at small aspect ratios (a/c<0.5) and smaller for a/c=1 as shown in Figure. 13. At the free surface maximum values developed for (a/c=1) and minimum occurred for small (a/c) as shown in Figure. 14.

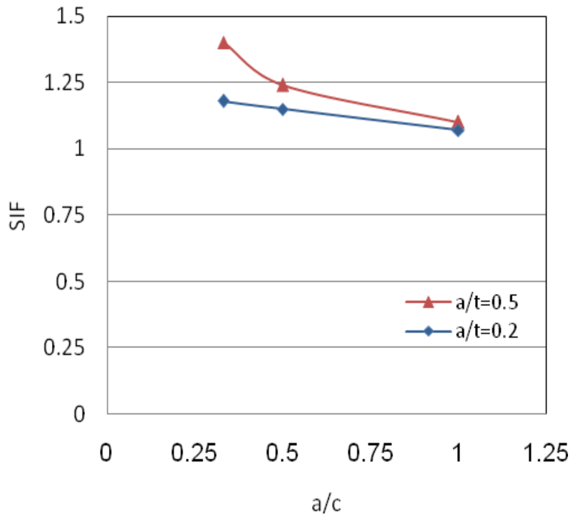

Figure 13. Stress IFntensity factor at the Deepest Point of Deep and Shallow Semi-Elliptical Surface Cracks (a/t=0.5, 0.2), as a Function of the Aspect Ratio a/c in Tension.

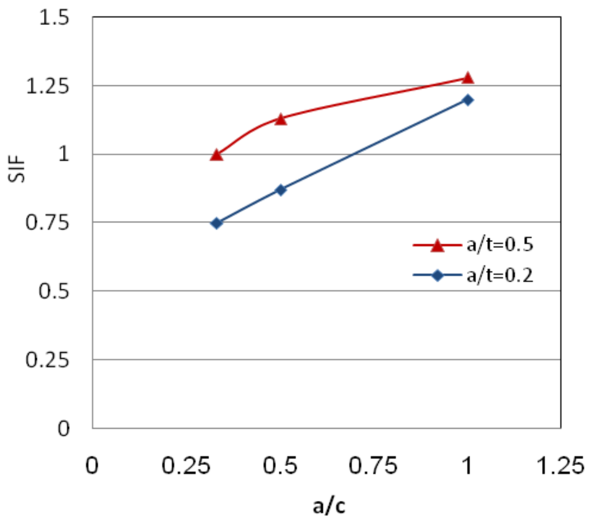

Figure 14. Stress Intensity Factor at the Free Surface of Deep and Shallow Semi-Elliptical Surface Cracks (a/t=0.5, 0.2), as a Function of the Aspect Ratio a/c in Bending.

The prediction of crack growth under elastic condition has been made based on the SIF determined. Figures 15 and 16 illustrate predictions of the crack shape development for shallow semi-elliptical surface cracks (a/t=0.2, a/c=0.33) under elastic conditions in bending and tension loadings. It can be seen that the crack growth in bending is different from that in tension. In bending the maximum predicted growth occurs at the deepest point as long as (a/t<0.5), however, as a/t ratio goes beyond (0.5) the maximum growth location on crack front shifted from deepest point to the free surface point as shown in Fig. 15. In tension, the crack development deviates from the semi-elliptical shape and acquires its semi-circular shape as the crack advances towards the wall thickness as shown in Figure. 16.

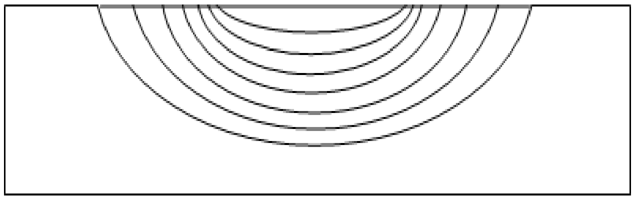

Figure 16. Schematic Representation of Crack Front Development in a Shallow Semi-Elliptical Surface Crack (a/t=0.2, a/c=0.33) Under Elastic Solution in Bending Loading.

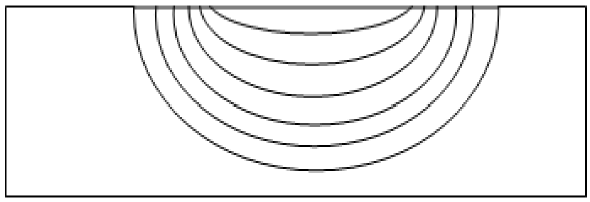

Figure 17. Schematic Representation of Crack Front Development in a Shallow Semi-Elliptical Surface Crack (a/t=0.2, a/c=0.33) Under Elastic Solution in Tension Loading.

VI. CONCLUSIONS

Maximum stress intensity factors for deep cracks in tension appear at the deepest point of the crack, while in bending the maximum occurs at the free surface. However, the maximum point changes over depending on the aspect ratio of the crack a/c. It is emphasized that it is generally difficult to obtain smooth distributions of stress intensity factors along the crack front due to the effect of a corner point singularity. In tension loading, the maximum and minimum SIF are always at the deepest point and the surface point, respectively, as long as (a/c<1). The large difference of SIF between these two points drives the crack to grow in the depth direction with a greater rate than in the width direction. Thus, surface cracks in tension are predicted to break the wall with a relatively small amount of crack growth in the width direction in comparison to bending where they break through with significant growth at surface.

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