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Design of Lead, Lag and Lead-Lag Compensators Based on Frequency Response Approach

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Abstract—**In the control systems, the compensators are widely used to improve system dynamics, characteristics of the open loop system and the stability. The degree of convergence of the output waveform for any compensator depends upon the proper selection and tuning of the compensator. The dynamic structure and the control properties of lead, lag and lead-lag compensators are presented in this work. A simple method based on the frequency domain is adopted to design the three different compensators. The parameters which have effect on each compensator performance were studied and explained how to choose them to achieve the optimal performance. The compensator algorithms are implemented with MATLAB simulation in order to control the DC motor motion. The results and the comparative analysis confirm the effectiveness of the proposed three compensators and improve the system response.**

Index Terms—**Bode diagram, DC motor, Lag compensator, Lead compensator, Lead-lag compensator.**

I. INTRODUCTION

The main task is to design a suitable controller that gives the required performance specifications such as rise time, settling time, maximum overshoot, gain margin, and phase margin. For this purpose, the well-known classical controller structures, namely P, PI, PID [1-2]. For greater accuracy of a system, steady state error should be small, but to reduce the steady state error the gain of the amplifier must be increased, overshoot will also increase and stability will decrease. However, there are interested in both accuracy and stability which can be done by connecting a circuit between error detector and plant, known as compensation [3].

The Compensators are corrective sub-systems introduced into the system to compensate for the deficiency in the performance of the plant. Therefore, given a plant and a set into the system to compensate for the deficiency in the performance of the plant. Therefore, given a plant and a set of specifications, suitable compensators can be designed, so that the overall system will meet the given specification [4].

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The lead and lag compensators are the most commonly utilized control architectures for designing control systems with the root locus (time domain) or bode (frequency domain) compensation method used. In [5], the cascade or series compensation is stated as the most popular method in the control system design. The design and implementation of the lead and lag compensators have been studied in [6-9]. These compensators have positive effects on different parts of the system performance. It can be summarized as lead compensator doesn't affect the steady state response while improving the transient response, and a lag compensator improves the steady state response and however slowing down the transient response. Therefore, using the two together is considered as a solution to improve the system transient and steady state responses [10]. A lead-lag compensator combines the advantages of both the lead and lag compensators [11]. The lead-lag compensator is used when both fast response and accuracy are required. By use of this compensator, the low-frequency gain can be increased which means an improvement in steady-state accuracy, and the system bandwidth and stability margins can be increased [12]. If the compensator is added, a bode diagram of the compensator can be simply added to the original bode diagram, and thus plotting the complete bode diagram is a simple matter. In addition, if the open-loop gain is varied, the magnitude curve shifted up or down without changing the slope of the curve, and the phase curve remains the same. For design purposes, therefore, it is simple to work with the bode diagram [12]. The design process of the lead-lag compensator has been introduced in [13-19]. A comparative analysis of lead, lag, lead-lag compensators with other different control techniques is studied in [20-22].

The work described in this paper focuses on the comparative analysis of lead, lag and lead-lag compensators and their design process. The design process is performed by applying the principles of the bode technique, using MATLAB simulation. This paper is organized as follows: section II presents the scheme of control system with cascade compensation. Section III describes the overall of the lead, lag and lead-lag compensators. Section IV shows the mathematical model of the simulation system which consists of a DC motor driving a rotational mechanical load. Section V illustrates the procedure of the proposed design method of the three different compensators. Section VI shows the simulation results and the comparative analysis which has been carried out on the basis of different parameters such as rise time, overshoot, and settling time. Finally, Section VII concludes the paper.

II. Closed Loop Control System Via Cascade Compensation

Fig. 1 shows the block diagram for designing the closed loop feedback system with lead, lag and lead-lag compensators using series connection. This design will be used to improve the Response of the DC motor driving a rotational mechanical load, which used as a plant in this paper.

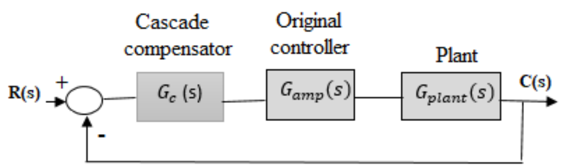

Figure. 1: Closed loop control system with cascade compensator

III. Lead, Lag And Lead-Lag Compensators

In phase lead compensation, alters the frequency response by adding a positive (leading) phase angle and, therefore, increases the phase margin at the crossover ($0dB$) frequency. Referring to the passive lead network of Fig. 2, its transfer function is given by [3]:

$$G_{lead}(s) = \frac{1+\alpha s T_1}{1+sT_1} = \alpha\left(\frac{s+\frac{1}{\alpha T_1}}{s+\frac{1}{T_1}}\right) \tag{1}$$

where

$$T_1 = \frac{R_1 R_2}{R_1+R_2} C \tag{2}$$

$$\alpha = \frac{R_1+R_2}{R_2} \quad \text{and} \quad \alpha > 1 \tag{3}$$

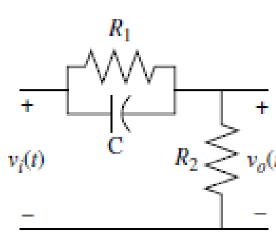

Figure. 2: Passive circuit of lead compensator

Note that the gain of α in Eq. (1) was added to eliminate an attenuation in the compensator's output. Furthermore, the transfer function has pole at $s = -1/T_1$ and zero at $s = -1/(\alpha T_1)$. This means the zero will be nearer to the origin in s-plane.

For phase lag compensation, the gain crossover frequency shifts to a lower value and, therefore, the bandwidth decreases and the response speed reduces, but the state error improves. Fig. 3 illustrates the passive lag compensator circuit.

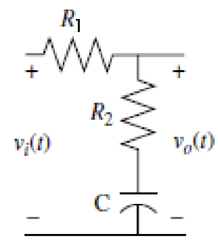

Figure. 3: Passive circuit of lag compensator

The transfer function can be written as [3]:

$$G_{lag}(s) = \frac{1+\beta T_2 s}{1+T_2 s} = \beta\left(\frac{s+\frac{1}{\beta T_2}}{s+\frac{1}{T_2}}\right) \tag{4}$$

where

$$T_2 = (R_1 + R_2)C \tag{5}$$

$$\beta = \frac{R_2}{R_1+R_2} \quad \text{and} \quad \beta < 1 \tag{6}$$

From Eq. (4), the transfer function has pole at $s = -1/T_2$ and zero at $s = -1/(\beta T_2)$. This means the pole is closer to the origin in s-plane.

The passive circuit of lead-lag compensation is shown in Fig. 4.

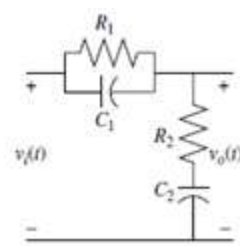

Figure. 4: Passive circuit of lead-lag compensator

This circuit looks like both compensators are cascaded. Therefore, the transfer function will be the product of transfer functions of Eqs (1) and (4) as following [3]:

$$G_{laglead}(s) = \alpha\beta\left(\frac{s+\frac{1}{\alpha T_1}}{s+\frac{1}{T_1}}\right)\left(\frac{s+\frac{1}{\beta T_2}}{s+\frac{1}{T_2}}\right) \tag{7}$$

where

$$\alpha T_1 = R_1 C_1 \tag{8}$$

$$\beta T_2 = R_2 C_2 \tag{9}$$

$$T_1 T_2 = R_1 R_2 C_1 C_2 \tag{10}$$

$$\alpha\beta = 1 \tag{11}$$

The first term in parentheses produces the lead compensation, and the second term in parentheses produces the lag compensation [10].

IV. System Model And Description

In order to investigate the proposed technique to design the three different compensators, a hybrid system of electrical and mechanical variables as shown in Fig. 5, are used. This system consists of a DC motor driving a rotational mechanical load with gears.

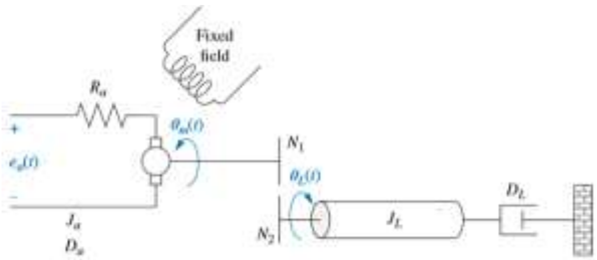

Figure.5: DC motor driving a rotational mechanical load

The desired transfer function between the output displacement $\theta_m(s)$ and input voltage $E_a(s)$ is found to be:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s+\frac{1}{J_m}\left(D_m+\frac{K_t K_b}{R_a}\right)\right]} \tag{12}$$

and

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 \tag{13}$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 \tag{14}$$

where K_b is a back emf constant, K_t is a motor torque constant, D_a is a motor damping, D_m is the equivalent damping at the motor side, D_L is a load damping, J_a is a motor inertia, J_m is the equivalent inertia at the motor side, J_L is a load inertia, $N_1{:}N_2$ is the gear ratio, R_a is a armature resistance. Consider the following values for the physical parameters:

Table 1: System Physical Parameters

Parameter	Value
K_b	$1\ Vs/rad$
K_t	$10\ Nm$
D_a	$2\ Nms/rad$
D_L	$800\ Nms/rad$
J_a	$5\ kgm^2$
J_L	$500\ kgm^2$
$N_1{:}N_2$	$1{:}10$
R_a	$1\ \Omega$

The system transfer function becomes:

$$G(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{1}{s(s+2)} \tag{15}$$

This transfer function is a second-order system of type 1, and the relationship between phase margin and percent overshoot as well as the relationship between closed-loop bandwidth and other time-domain specifications were derived [11].

V. Design Procegures

The design procedures for the three different compensators, based on the frequency response approach, are stated as follows [11]:

1) Set the DC gain of the uncompensated system to meet the steady state error requirement.
2) Draw the bode diagram for the uncompensated system and determine the current phase margin.
3) Calculate the phase margin (ϕ_{req}) required to meet the damping ratio (ζ) as follows:

$$\phi_{req} = tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} \tag{16}$$

4) Determine the closed loop bandwidth (ω_{BW}) required to meet the transient performance requirement, such as settling time (t_s) or peak time (t_p), using the equation given below:

$$\omega_{BW} = \omega_n\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}} \tag{17}$$

where ω_n is a natural frequency and can be computed by:

$$\omega_n = \frac{4}{\zeta t_s} = \frac{\pi}{t_p\sqrt{1-\zeta^2}} \tag{18}$$

5) Select a new phase margin frequency (ω_{new}) that is slightly less than the bandwidth. At this frequency, determine the available phase-margin ($\phi_{\omega_{new}}$).
6) With adding some phase margin (ε) which is known as a margin of safety, the maximum phase margin (ϕ_m) can be determined by:

$$\phi_m = \phi_{req} - \phi_{\omega_{new}} + \varepsilon \tag{19}$$

7) Determine β value from the following equation:

$$\beta = \frac{1-sin\phi_m}{1+sin\phi_m} \tag{20}$$

8) For the lag compensator in Eq (4), use the equation given below to find the parameter value of T_2:

$$T_2 = \frac{10}{\beta\omega_{new}} \tag{21}$$

9) For the lead compensator in Eq. (1), calculate the parameters α and T_1 values as follows:

$$\alpha = \frac{1}{\beta} \tag{22}$$

$$T_1 = \frac{\sqrt{\beta}}{\omega_{new}} \tag{23}$$

10) To obtain the transfer functions of the lead, lag and lead-lag compensators, substitute the calculated values of α, β, T_1 and T_2 in Eqs. (1) (4) and (7). Simulate the compensated system and repeat the design as necessary.

VI. Simulation And Results

The proposed compensators in this paper has been validated on simulation environment. The MATLAB programming was used to implement the transfer function of the system, design the compensators, and study the design effectiveness on the system response. The presented system here consists of DC motor driven mechanical load and operational amplifier. The open loop transfer function of the uncompensated system is:

$$G_{sys}(s) = \frac{K}{s(s+2)(s+20)} \quad (24)$$

where K is the amplifier gain. The designed system should achieve the following specifications:

Table 2: Design Specifications

Parameter	Value
Static velocity error constant (K_v)	37.50
Peak time (t_p)	$\leq 2.00\ s$
Overshoot (os)	15.00 %

Based on the given velocity error constant in Table 2, The K value was determined and found to be:

$$K = 1500 \quad (25)$$

Therefore, the transfer function of uncompensated system in Eq. (24) becomes:

$$G_{sys}(s) = \frac{1500}{s(s+2)(s+20)} \quad (26)$$

The bode diagram of the uncompensated open loop system with adjusted gain is presented in Fig. 6. Note that the closed loop of uncompensated system is unstable.

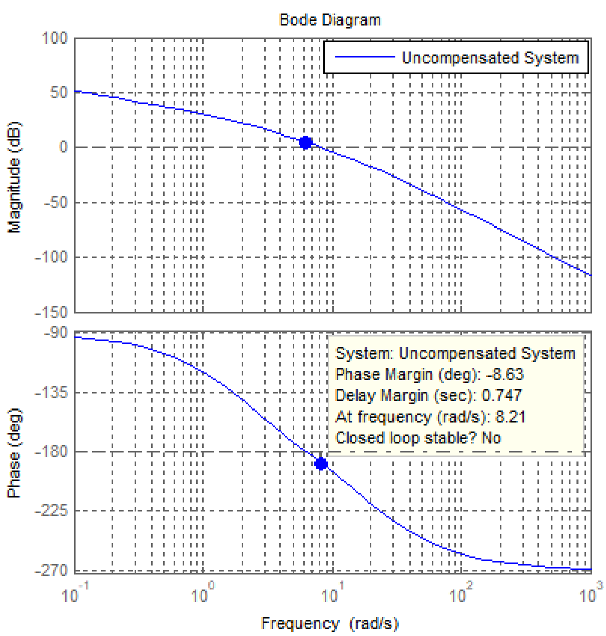

Figure. 6: Bode diagram for uncompensated systems

The other parameter values that are necessary to achieve the design, and meet the desired requirements, were calculated and illustrated in Table 3.

Table 3: Details of design parameters

Parameter	Value
ζ	0.5170
ω_{BW}	$6.4739\ rad/s$
ϕ_{req}	53.1718^o
$\phi_{\omega_{new}}$	6.60^o
ϕ_m	51.5718^o
T_1	$0.0673\ s$
T_2	$15.9007\ s$
α	8.2346
β	0.1214

From these values, the transfer functions of designed compensators will be in the following forms:

$$G_{lead}(s) = 8.2346\frac{(s+1.8050)}{(s+14.8682)} \quad (27)$$

$$G_{lag}(s) = 0.1214\frac{(s+0.5179)}{(s+0.06289)} \quad (28)$$

$$G_{lead-Lag}(s) = \frac{(s+0.5179)(s+1.805)}{(s+0.06289)(s+14.86)} \quad (29)$$

As the different compensators were designed, each one is employed in the forward path of the system and the bode diagram of the compensated system is displayed in Fig. 7. It can be seen that the lag compensator increases the low frequency gain, and reduces the high frequency gain and the gain crossover frequency of the system. Meanwhile, the lead compensator allows the high gain at high frequencies and low gain at low frequencies, and rises the system gain crossover frequency. However, the lead-lag compensator gives high gain at all frequencies of the system and increases the gain crossover frequency. The frequency response characteristics of the three compensated systems are evaluated and presented in Table 4.

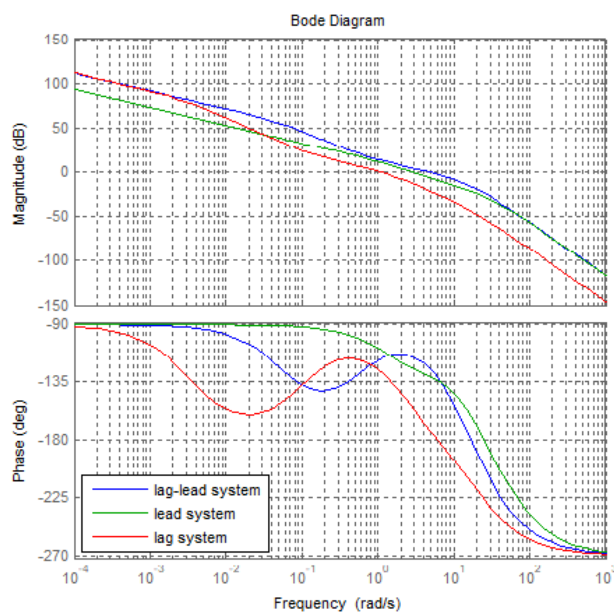

Figure. 7: Bode diagrams for lead, lag and lead-lag compensated systems

Table 4: System frequency response characteristics

Parameter	Lead compensator	Lag compensator	Lead-Lag compensator
Phase margin (PM)	56.60^o	53.36^o	56.00^o
Phase margin frequency	$2.98\ rad/s$	$0.95\ rad/s$	$4.65\ rad/s$
Closed loop bandwidth (os)	$5.31\ rad/s$	$1.81\ rad/s$	$9.80\ rad/s$

Fig. 8 illustrates the time response comparison between the lead, lag and lead-lag compensators when incorporated with the DC motor system in a closed loop. It can be easily seen that the three closed loop compensated systems are stable and their responses are significantly improved compared with the uncompensated system. Adding the lag compensator improves the steady state error of the system. However, it does not contribute much to the time response required. The lead compensator provides a much faster response as compared to the lag controller as expected but still does not meet the higher time response requirements. The lead-lag compensator provides both faster response and steady state error reduction and therefore meets the requirements as desired. Furthermore, a comparative analysis between designed compensators in terms of different time domain specifications are shown in Table 5.

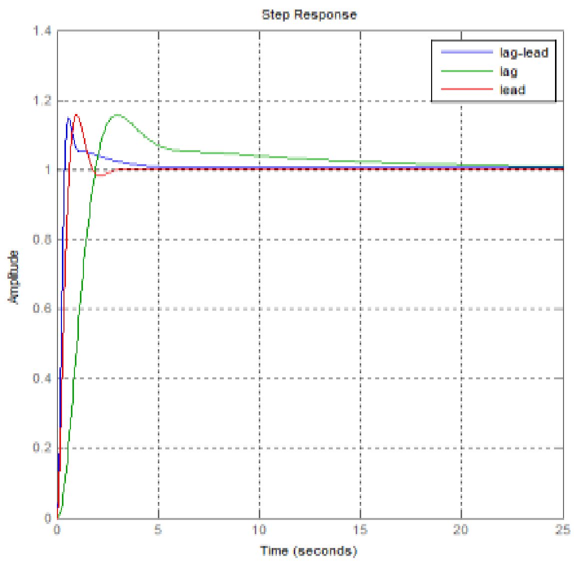

Figure. 8: Step response of lead, lag and lead-lag compensated systems

Table 5: System time response characteristics

Parameter	Lead compensator	Lag compensator	Lead-Lag compensator
Static velocity error constant (K_v)	37.496	38.121	37.511
Steady state error (e_{ss})	2.667 %	2.623 %	2.666 %
Overshoot (os)	15.80 %	15.70 %	15.00 %
Peak time (t_p)	0.984 s	2.990 s	0.583 s
Rise time (t_r)	0.422 s	1.270 s	0.245 s
Settling time (t_s)	1.660 s	16.500 s	3.380 s

As can be seen in Table 5, it is clearly that the response of the lead-lag compensated system is faster and better than the other, due to shifted gain crossover frequency to higher value and increased the bandwidth, accuracy is improved, and all the design specifications are successfully met. The reduced bandwidth in the lag compensated system lead to the slow response, where peak time, rise time and settling time are increased. The system with lead compensation has increased the phase margin and bandwidth and the response has become fast, but the maximum overshoot still exceeds the target. For steady state error, the lag compensator has more influence than lead and lead-lag compensators.

VII. Conclusion

This paper has demonstrated the design of the lead, lag and lead-lag compensators using frequency domain techniques (bode diagram) is described, followed by the detailed analysis of the various components used in the system and the design steps of the compensators are explained. A set of simulated results have been obtained from a mathematical model of DC motor driven mechanical load, and has been clearly shown that:

- All the design specifications are successfully met with the lead-lag compensator.
- The transient response improved with the lead compensator with little exceeded in the target of the maximum overshoot.
- The steady state decreased with the lag compensator but the system response has become slow.
- This technique is simple and attractive and could easily be extended to design the compensators in embedded systems.

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