Design and Simulation of Torque Control for Hot Rolling Steel Mill

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Abstract— **Lateral movement of a strip in the hot rolling process is an important unsolved technical problem. To minimize the strip thickness variations, the state space model formulation is usually used. It results in reducing productivity and, in the extreme case, strip tearing during tailing out at the finishing mill. This movement is caused by various asymmetric factors such as mechanical asymmetry of the mill and different temperatures along the strip width direction. Furthermore, the difference in the temperature and the thickness of steel strip resulting in deformation of the steel, which affects the bending torque of the steel strip. This study aims to understand the influences of these various factors on lateral movement. In order to reduce the lateral movement and the disturbance terms, that act on uncertain inter-connected systems, a new control approach, based on a combination of Integral Sliding Mode Control (ISMC) and Linear Matrix Inequalities (LMI) technique, is proposed. Simulation results of hot rolling system with three interconnected systems are presented to validate the new controller performance.**

Index Terms: **Hot rolling, Decentralized control, Integral sliding mode control.**

I. INTRODUCTION

The rolling mill process consists of introducing a strip inside two rotating rolls causing a permanent deformation in this strip, it is called thickness reduction. The stands, and rotary rolls, are the machines that make the rolling mill process. The hot-rolling process has gradually become a much more closely controlled operation, and is being increasingly applied to low alloy steels with compositions carefully chosen to provide optimum mechanical properties when the hot deformation is complete. The main motivation of this work is the reduction of the output thickness variations of any of two input process variables disturbance, strip temperature and strip thickness. In more highly alloy steels, it is possible to subject the steels to heavy deformations in the austenitic conditions prior to transformation. This process, auto forming, allows the attainment of very high strength levels combined with good toughness[1]. The current state of automation and control for hot rolling mills, with a particular emphasis on flat products, is introduced.

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A typical modern control–system design is discussed together with the impact on the design of process models, available measurements, available control actuators and material properties. Currently, control of microstructure is achieved by imposing deformation and temperature time profiles on the process. The need for more stringent product quality will lead to the demand for a tighter control of microstructure.

The novelty of this paper is its combination of ISMC and LMI technique for torque control of hot rolling steel mill system, verified by simulation study. The rest of the paper is organized as follows: section II presents the problem formulation. Section III describes the design procedures of the new control approach that includes ISMC in the first part and LMI control in the second part. The overall simulation system is illustrated in section IV. The simulation results and discussion are shown in section V. The improvement in the system performance with the proposed control method has been demonstrated in comparison with no controller and LMI controller. It will demonstrate that the proposed controller can reliably regulate the system response, regardless of actuator fault severity, disturbance and uncertainties.

II. PROBLEM FORMULATION

Consider an inter-connected system comprising subsystems described by:

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + Z_i(x_i,t) + W_i(x_i,t) \\ &\quad + E_i d_i(t) + B_i f_i(t) \\ y_i(t) &= C_i x_i(t) \\ i &= 1,2,..,n\end{aligned} \tag{1}$$

$x_i(t) \in R^n$ is the state vector, $u_i(t) \in R^m$ is the control input vector and $y_i(t) \in R^p$ is the system output vector. $A_i \in R^{n\times n}$ is the subsystem characteristic matrix, $B_i \in R^{n\times m}$ is the subsystem control input matrix, $C_i \in R^{p\times n}$ is the subsystem output matrix and $E_i \in R^{n\times q}$ is the subsystem external disturbance matrix, all these matrices are known. $Z_i(x_i,t)$ denotes the interactions between subsystems and $W_i(x_i,t)$ denotes the uncertainties where $W_i(x_i,t) = B_i Q_i(x_i,t)$ is unknown. $d_i(t)$ represents the unknown bounded disturbance and $f_i(t) \in R^k$ represents the actuator faults where $f_i(t) = -K(t)u_i(t)$, $K(t) = \mathrm{diag}(K_i)$ and $0 \le K_i \le 1$. $K_i = 0$ means the actuator is working perfectly while $K_i = 1$ means the actuator has failed completely, otherwise the fault is present. The following assumptions are taken into account in this study[2]:

- A1. The pair (A_i, B_i) is controllable.
- A2. The B_i has full rank m_i.
- A3. The initial state $x_i(t_0)$ is bounded.
- A4. $Z_i(x_i, t)$ is Euclidean bounded norms as: $\|Z_i(x_i, t)\| \leq \beta_i(x_i, t)$ where $\beta_i(x_i, t)$ is known nonlinear function[4].
- A5. $Q_i(x_i, t)$ is bounded as: $\|Q_i(x_i)\| \leq \kappa_i\|x_i\|$ where $\kappa_i > 0$ is known Lipschitz constants[5].
- A6. $d_i(t)$ is Euclidean bounded norms as: $\|d_i(t)\| \leq \gamma_i\|x_i\|$ where $\gamma_i > 0$ is known.
- A7. $f_i(t)$ is Euclidean bounded norms as: $\|f_i(t)\| \leq \eta_i\|x_i\|$ where $\eta_i > 0$ is known Lipschitz constants.

Suppose that: $\Gamma = I_n\text{-}BB^+$ where B^+ is pseudo-inverse of the matrix B, $B^+ = (B^TB)^{-1}B^T$ and I_n is the $n \times n$ identity matrix. Interactions between subsystems Z_i contain two components; matched Z_{mi} and unmatched Z_{ui} spaces[6], and therefore it can computed by $Z_i = Z_{mi} + Z_{ui}$ where $Z_{mi} = B_iB_i^+Z_i$ and $Z_{ui} = \Gamma_iZ_i$. The same procedure is applied for the compound E_id_i which can be obtained by $E_id_i = E_id_{mi} + E_id_{ui}$ where $E_id_{mi} = B_iB_i^+E_id_i$ and $E_id_{ui} = \Gamma_iE_id_i$. After substituting all assumptions, the i^{th}subsystem will be:

$$\begin{aligned} \dot{x}_i(t) &= A_ix_i(t) + B_iu_i(t) + B_i\Phi_{mi}(t) + \Phi_{ui}(t) \\ y_i(t) &= C_ix_i(t) \\ i &= 1,2,\dots\dots,n \end{aligned} \tag{2}$$

where Φ_{mi} and Φ_{ui} are the matched and unmatched components, respectively, and can be computed by:

$$\begin{aligned} \Phi_{mi} &= B_i^+Z_i(t) + Q_i(x_i, t) + B_i^+E_id_i(t) + f_i(t) \\ \Phi_{ui} &= Z_{ui}(t) + Ed_{ui} = [\Gamma_i\Gamma_iE_i]\begin{bmatrix} Z_i \\ d_i \end{bmatrix} = r_iw_i \end{aligned} \tag{3}$$

The control signal contains two components as:

$$u_i(t) = u_i^{LMI}(t) + u_i^{ISM}(t) \tag{4}$$

where u_i^{LMI} is responsible for stabilizing the system, obtaining the desired performance and decreasing the effects of unmatched components, u_i^{ISM} is the discontinuous control signal which is responsible for rejecting the effects of matched components (uncertainties, disturbances and actuator faults). Substituting (4) into (3) gives the i^{th} subsystem equation, including the ISMC, as:

$$\begin{aligned} \dot{x}_i(t) = A_ix_i(t) &+ B_iu_i^{LMI}(t) + B_iu_i^{ISM}(t) \\ &+ B_iB_i^+Z_i(t) + \zeta_i(t) + B_iQ_i(x_i, t) \\ &+ E_id_i(t) + B_if_i(t) \\ y_i(t) = C_ix_i(t)& \\ i = 1,2,\dots\dots,n& \end{aligned} \tag{5}$$

III. CONTROL DESIGN

A. ISMC Design

The design of the control system based on the inter-connected systems depends on knowledge of the type and extent of the subsystem interconnections. It is assumed that all the subsystems are connected to each other and the i^{th} subsystem interconnections Z_i is unknown, this implies the distribution matrices of the interconnections are also unknown. The de-centralized subsystem control system corresponding to this scenario is now developed using the classical ISMC approach. The ISMC can be achieved through the following steps:

1. Design a sliding surface to satisfy a chosen linear system performance specification when the system is on the sliding surface.
2. Design an appropriate discontinuous control to maintain the chosen sliding motion. The integral sliding switching surface is proposed as:

$$\sigma_i(x_i, t) = G_i[x_i(t) - x_i(t_o) - \int_{t_o}^{t}\left(A_ix_i(t) + B_iu_i^{LMI}(t)\right)]dt \tag{6}$$

where G_i is the appropriate design matrix which must satisfy the condition that G_iB_i is invertible if the actuator has not failed completely. The integral term provides the freedom to add any linear controller that satisfies the prescribed time response performance specifications. The structure of (6) implies that there is independent freedom to choose any control design method of the linear component of the feedback. This is an important characteristic of the ISMC design problem. The so-called equivalent control $u_{eqi}(t)$ can maintain the state motion on the sliding surface if the actuator fault is bounded by letting the time derivative of $\sigma_i(x_i, t)$ be identically zero, i.e[7].

$$\dot{\sigma}_i(x_i, t) = G_i\dot{x}_i(t) - G_iA_ix_i(t) - G_iB_iu_i^{LMI}(t) = 0 \tag{7}$$

Substituting (5) into (7) yields:

$$\begin{aligned} G_iA_ix_i + G_iB_iu_i^{LMI} + G_iB_iu_i^{ISM} &+ G_iB_iB_i^+Z_i(t) \\ &+ G_i\zeta_i(t) + G_iB_iQ_i(x_i, t) \\ &+ G_iE_id_i(t) + G_iB_if_i(t) - G_iA_ix_i \\ &- G_iB_iu_i^{LMI} = 0 \end{aligned} \tag{8}$$

This leads to the equivalent control signal if $f_i(t)$ is not failure ($K_i \neq 0$) as:

$$\begin{aligned} u_{eqi}(t) = -(G_iB_i)^{-1}[&G_iB_iB_i^+Z_i(t) + G_i\zeta_i(t) \\ &+ G_iB_iQ_i(x_i, t) + G_iE_id_i(t) \\ &+ G_iB_if_i(t)] \end{aligned} \tag{9}$$

Equation (9) can be rewritten as:

$$\begin{aligned} u_{eqi}(t) = -B_i^+Z_i(t) &- (G_iB_i)^{-1}G_i\zeta_i(t) - Q_i(x_i, t) \\ &- (G_iB_i)^{-1}G_iE_id_i(t) - f_i(t) \end{aligned} \tag{10}$$

From (5) and (10), the equivalent dynamic equation of the i^{th} subsystem in sliding mode becomes:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + [I_i - B_i(G_iB_i)^{-1}G_i]\zeta_i(t) + [I_i(G_iB_i)^{-1}G_i]E_i d_i(t) \tag{11}$$

From (1), the unknown matched uncertainties $(G_iB_i)^{-1}G_i\zeta_i(t)$ and actuator faults $f_i(t)$ are completely nullified. However, the dynamics of the subsystem on the sliding surface still contain the unknown unmatched uncertainties and disturbances $[I_i\text{-}B_i(G_iB_i)^{-1}G_i]\zeta_i(t)$. The proposed discontinuous control takes the following form:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\|} \tag{12}$$

It is assumed that μ_i is a positive scalar function, and according to the stability of the subsystem, a possible choice is:

$$\mu_i > \|(G_iB_i)^{-1}G_i\|\beta_i(x_i,t) + \kappa_i\|x_i\| + \eta_i\|x_i\| + \gamma_i\|(G_iB_i)^{-1}G_iE_i\|\|x_i\| \tag{13}$$

To maintain the subsystem state on the sliding surface, let $\sigma_i(x_i,t) = 0$. Then, the subsystem stability is considered by choosing the following subsystem Lyapunov functions:

$$\sum_{i=1}^{N} V_i\left(\sigma_i(x_i,t)\right) = \sum_{i=1}^{N} \|\sigma_i(x_i,t)\| > 0 \tag{14}$$

For stability of each subsystem, the time derivatives of each $V_i(\sigma_i(x_i,t)$ must be negative, i.e. $\dot{V}_i(\sigma_i(x_i,t)) < 0$ this can be verified as follows:

$$\dot{V}_i(\sigma_i(x_i,t)) = \frac{\sigma_i^T(x_i,t)\dot{\sigma}_i(x_i,t)}{\|\sigma_i(x_i,t)\|} \tag{15}$$

with

$$\dot{\sigma}_i(x_i,t) = G_iB_iu_i^{ISM} + G_iZ_i(t) + G_iB_iQ_i(x_i,t) + G_iE_id_i(t) + G_iB_if_i(t) \tag{16}$$

Substituting the proposed discontinuous control in (12) into (16) and substituting the result into (15) yields:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) = \sum_{i=1}^{N}[-G_iB_i\mu_i + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}G_iZ_i(t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}G_iB_iQ_i(x_i,t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}G_iE_id_i(t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}G_iB_if_i(t)] \tag{17}$$

Equation (17) can be rearranged as:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \le \sum_{i=1}^{N} -(G_iB)[\mu_i - (G_iB_i)^{-1}G_i\|Z_i\| - \|Q_i\| - (G_iB_i)^{-1}G_iE_i\|d_i\| - \|f_i\|] \tag{18}$$

According to assumptions A4, A5, A6 and A7, (18) becomes:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \le \sum_{i=1}^{N} -(G_iB)\,[\mu_i - \|(G_iB_i)^{-1}G_i\|\beta_i(x_i,t) - \eta_i\|x_i\| - (G_iB_i)^{-1}G_iE_i\|d_i\| - \kappa_i\|x_i\| - \|f_i\|] - \gamma_i\|(G_iB_i)^{-1}G_iE_i\|\|x_i\|] \tag{19}$$

Furthermore, according to the choice in (12) that leads to $\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \le 0$. This means that the choice of the μ_i according to the inequality (13) indeed guarantee the stability of the sliding surface by using the proposed discontinuous control. The matrices G_i must be chosen in order to reduce the norm of $\Psi_i\zeta_i(t)$ and $\Psi_iE_id_i(t)$, as well as the amplification of Ψ_i to the unknown unmatched uncertainties and disturbances[8]. By substituting the matrices $B_i^+$of G_i in (11) and then some terms can be rewritten as:

$$[I_i - B_i(G_iB_i)^{-1}G_i]\zeta_i(t) = [I_i - B_i(B_i^+B_i)^{-1}B_i^+]B_i^\perp B_i^{\perp+}Z_i(t) \tag{20}$$

where $\zeta_i(t) = B_i^\perp B_i^{\perp+}Z_i(t)$ since $B_i^TB_i^\perp = 0$, these terms can be rewritten as:

$$[I_i - B_i(B_i^+B_i)^{-1}B_i^+]B_i^\perp B_i^{\perp+}Z_i(t) = B_i^\perp B_i^{\perp+}Z_i(t) \tag{21}$$

Also,

$$[I_i - B_i(G_iB_i)^{-1}G_i]E_id_i(t) = [I_i - B_i(B_i^+B_i)^{-1}B_i^+]E_id_i(t) = [I_i - B_iB_i^+]E_id_i(t) \tag{22}$$

Combining (11), (21) and (22), this leads to:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{LMI}(t) + B_i^\perp B_i^{\perp+}Z_i(t) + [I_i - B_iB_i^+]E_id_i(t) \tag{23}$$

The dynamics of the subsystems on the sliding surfaces can thus be described as:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{LMI}(t) + T_iZ_i(t) + M_id_i(t) \tag{24}$$

where $T_i = B_i^\perp B_i^{\perp+}$ and $M_i = [I_i - B_iB_i^+]E_i$. From (24) it can be observed that the unknown unmatched uncertainties and disturbances have not been completely eliminated. As a result, another method must be found to enhance the properties of the control design to reduce the influences of the unknown unmatched uncertainties and disturbances. Note that, applying the $u_i^{ISM}(t)$ to the subsystems, so-called chattering motion takes place, i.e. repeated discontinuous motion in a small vicinity of each sliding surface is present. The chattering motion can be reduced by adding a small constant $Z_i > 0$ if all uncertainties disappear and the subsystem is stable, where the control will be as[9].

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i} \tag{25}$$

B. LMI Control Design

This procedure aims to design a de-centralized control that robustly regulates the state of the overall system without any information exchange between the controllers. On other hand each de-centralized control uses only available local information. After the ISMC is designed for the i^{th} subsystem, the dynamic i^{th} subsystem is described by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + \Gamma_i J_i(t) \tag{26}$$

where $\Gamma_i = [T_i M_i]$ and $J_i(t) = \begin{bmatrix} Z_i(t) \\ d_i(t) \end{bmatrix}$. Suppose $J_i(t)$ is an unknown input but conforms to the quadratic inequality condition.

$$J_i^T(t) J_i(t) \le \alpha_i^2 x_i^T(t) x_i(t) \tag{27}$$

where the $\alpha_i > 0$ is scalar parameter. Then the overall (one shot) system can be written as:

$$\dot{X}(t) = A_d X(t) + B_d U(t) + \Gamma_d J(t) \tag{28}$$

where $X(t) = [x_1, x_2, \dots, x_n]$, $U(t) = [u_1^{LMI}, u_2^{LMI}, \dots, u_n^{LMI}]$, $J(t) = [J_1, J_2, \dots, J_n]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $\Gamma_d = diag(\Gamma_i)$, where $diag$ is a block diagonal matrix. To develop a robust control law, let the feedback take the following form:

$$U(t) = KX(t) \tag{29}$$

where K is the controller gain. The purpose of choosing K value is to minimize the effect of $J(t)$ on the one shot system. The unknown input disturbance satisfies the condition of the quadratic inequality:

$$J^T(t) J(t) \le \alpha^2 X^T(t) X(t) \tag{30}$$

where $\alpha > 0$ is a scalar parameter. To check the stability of this one shot closed-loop system choose a candidate Lyapunov function candidate $V(X,t) = X^T(t) P X(t)$, where $P = diag(P_i)$ and $P_i > 0$ are s.p.d. matrices. The choice of control method depends on the designer and can be either de-centralized control by choosing P as a diagonal matrix or by de-centralized overlapping control by choosing P as a non-diagonal matrix. The time derivatives of $V(X,t)$ is given by:

$$\dot{V}(X,t) = \dot{X}^T(t) P X(t) + X^T(t) P \dot{X}(t) \tag{31}$$

The set of (26), (29) and (31) results in:

$$\begin{aligned} \dot{V}(X,t) = {} & [A_d X(t) + B_d U(t) + \Gamma_d J(t)]^T P X(t) \\ & + X^T(t) P [A_d X(t) + B_d U(t) \\ & + \Gamma_d J(t)] \end{aligned} \tag{32}$$

Re-arranging (32) gives:

$$\begin{aligned} \dot{V}(X,t) = {} & X^T(t) A_d^T P X(t) + X^T K^T B_d^T P X(t) \\ & + J^T \Gamma_d^T P X(t) + X^T(t) P A X(t) \\ & + X^T(t) P B K X(t) \\ & + X^T(t) P \Gamma_d J(t) \end{aligned} \tag{33}$$

The stability of the system (26) requires that $\dot{V}(X,t) < 0$ for all $X(t) \neq 0$. Equation (32) can be rewritten as:

$$\mathcal{Z}^T \mathcal{D} \mathcal{Z} < 0 \tag{34}$$

where $\mathcal{D} = \begin{bmatrix} A_d^T P + P A_d + K^T B_d^T P + P B_d K & P\Gamma_d \\ \Gamma_d^T P & 0 \end{bmatrix}$ and $\mathcal{Z} = \begin{bmatrix} X(t) \\ J(t) \end{bmatrix}$. In order to check the condition stability, the matrix $\mathcal{D}$ must be negative-definite, i.e. $\mathcal{D} < 0$. Equation (30) could be rewritten as:

$$\mathcal{Z}^T \mathcal{O} \mathcal{Z} \le 0 \tag{35}$$

where $\mathcal{O} = \begin{bmatrix} -\alpha^2 I & 0 \\ 0 & I \end{bmatrix}$. Equations (34) and (35) can be combined into one single equation by using the S-procedure[10]. If $\mathcal{D}$ and $\mathcal{O}_i$ are symmetric matrices then $\mathcal{Z}_i^T \mathcal{D} \mathcal{Z} < 0$ and $\mathcal{Z}_i^T \mathcal{O}_i \mathcal{Z} \le 0$, where there is a number $\tau > 0$ that satisfies the relation $\mathcal{D} - \tau_i \mathcal{O}_i < 0$. Therefore the combination of the two equations is:

$$\begin{aligned} \mathcal{D} - \tau \mathcal{O} = {} & \begin{bmatrix} A_d^T P + P A_d + K^T B_d^T P + P B_d K & P\Gamma_d \\ \Gamma_d^T P & 0 \end{bmatrix} \\ & - \tau \begin{bmatrix} -\alpha^2 I & 0 \\ 0 & I \end{bmatrix} < 0 \end{aligned} \tag{36}$$

Put $Y = P/\tau$ in (36), this yields to

$$\begin{bmatrix} A_d^T \mathcal{Y} + \mathcal{Y} A_d + K^T B_d^T \mathcal{Y} + \mathcal{Y} B_d K + \alpha^2 I & \mathcal{Y}\Gamma_d \\ \Gamma_d^T \mathcal{Y} & -I \end{bmatrix} < 0 \tag{37}$$

Equation (37) cannot be solved by LMI due to the bilinear terms $\mathcal{Y} B_d K$. To overcome this limitation the non-convex problem must be converted into a convex problem. To achieve this, both sides of (37) must be multiplied by the matrices $\begin{bmatrix} \mathcal{Y}^{-1} & 0 \\ 0 & I \end{bmatrix}$ and $\mathcal{P} = \mathcal{Y}^{-1}$.

$$\begin{bmatrix} \mathcal{P} A_d^T + A_d \mathcal{P} + \mathcal{P} K^T B_d^T + B_d K \mathcal{P} + \alpha^2 \mathcal{P}\mathcal{P} & \Gamma_d \\ \Gamma_d^T & -I \end{bmatrix} < 0 \tag{38}$$

After using the S-procedure [11] and Schur complement then the inequality (38) can be re-formulated as:

$$\begin{bmatrix} \mathcal{P} A_d^T + A_d \mathcal{P} + N^T B_d^T + B_d N + \Upsilon^T 7 \Upsilon & \Gamma_d & \mathcal{P} \\ \Gamma_d^T & -I & 0 \\ \mathcal{P} & 0 & \epsilon I \end{bmatrix} < 0 \tag{39}$$

where Υ is a tuning matrix, $N = K\mathcal{P}$ and $\epsilon = 1/\alpha^2$.

Algorithm I:

1. Calculate $\sigma_i(x_i, t)$ from (6).
2. Obtain the discontinuous control signal by

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i, t)}{\|\sigma_i(x_i, t)\| + \mathfrak{z}_i}$$

3. Calculate the aggregate system from (26).
4. Minimize ϵ subject to $\mathcal{P} > 0$ and (39).
5. Determine the controller gain by $K = N\mathcal{P}^{-1}$.

Limiting the gain so that it is not too high by adding other conditions to the LMI algorithm. In addition, a condition can be added to the matrix N as:

$$\begin{bmatrix} -k_N I & N^T \\ N & -I \end{bmatrix} < 0 \tag{40}$$

As well as adding another condition to the matrix $\mathcal{P}$ as:

$$\begin{bmatrix} \mathcal{P} & I \\ I & k_P \mathrm{I} \end{bmatrix} > 0 \tag{41}$$

where k_N and k_P are scalar variables.

Algorithm II:

This algorithm depends on minimizing the variables ϵ, k_N and k_P subjecting to $\mathcal{P} > 0$ in (39), (40) and (41).

The control of overall system is achieved by using the ISMC and LMI gains. The P matrix is chosen to be a diagonal matrix to obtain the local controller. The control is attained by the combined ISMC and LMI to minimize the unmatched uncertainty, as shown in figure 1.

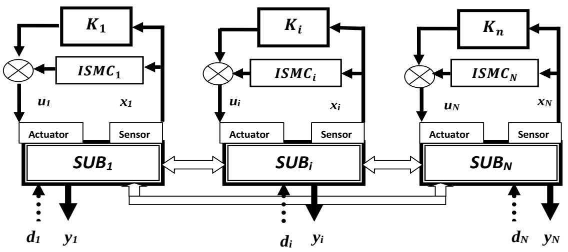

Figure 1. Control of inter-connected systems via ISMC and LMI

IV. NUMERICAL EXAMPLE

Consider the following hot rolling steel mill system, as shown in figure 2, which consists of three intercommoned non-linear systems adapted from [6].

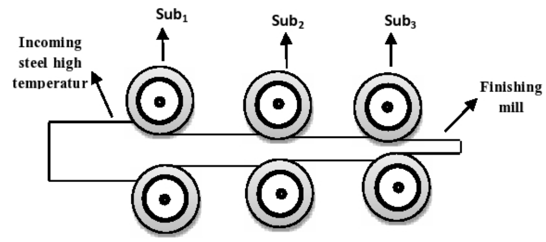

Figure 2. The hot rolling system

Subsystem$_1$ parameters:

$$A_1 = \begin{bmatrix} -430 & 4.8 \\ -640 & -97.5 \end{bmatrix} \quad B_1 = \begin{bmatrix} 3.9 \\ 0 \end{bmatrix} \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0.107 & -3.6 \\ 02.44 & -43.6 \end{bmatrix} \quad x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0 & 0 \\ 375 & -3.8 \end{bmatrix} \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

Subsystem$_2$ parameters:

$$A_2 = \begin{bmatrix} -176 & 2.52 \\ -630 & -125.6 \end{bmatrix} \quad B_2 = \begin{bmatrix} 2.5 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.026 & -0.73 \\ -2.37 & 65.2 \end{bmatrix} \quad x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 & -3 \\ 0 & -103.7 \end{bmatrix} \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 410 & -11 \end{bmatrix} \begin{bmatrix} x_{31}(t) \\ x_{32}(t) \end{bmatrix}$$

Subsystem$_3$ parameters:

$$A_3 = \begin{bmatrix} -4.052 & -184 \\ -630 & -31.1 \end{bmatrix} \quad B_3 = \begin{bmatrix} 3.7 \\ 0 \end{bmatrix} \quad C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} -0.0136 & 0.38 \\ 3 & -82 \end{bmatrix} \quad x_3(t) = \begin{bmatrix} x_{31}(t) \\ x_{32}(t) \end{bmatrix}$$

$$z_3 = \begin{bmatrix} 0 & -0.91 \\ 0 & 200 \end{bmatrix} \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 102.8 \end{bmatrix} \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

Disturbance signal:

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 10 \end{bmatrix}$$

where $x_{11}(t)$, $x_{21}(t)$ and $x_{31}(t)$ are stands for the angular velocity deviations from operating values of the three subsystems. $x_{12}(t)$, $x_{22}(t)$ and $x_{32}(t)$ are stands for the outlet tension deviations from the operating values of the three subsystems. $d_1(t)$ is the deviation of slab temperature from operating value at inlet of subsystem$_1$. $d_2(t)$ is the slab thickness at inlet of subsystem$_1$. $u_1(t)$ and $u_2(t)$ are stands for the armature voltage deviations from the operating values of the motors controlling the three subsystems. This mathematical model was built in the MATLAB environment by the authors to represent all the subsystem matrices as well as the controller.

V. SIMULATION RESULTS

In order to verify the proposed control method several tests were carried out on the system, described in section IV, using MATLAB environment under healthy and faulty actuator conditions. A 50% of torque was applied in both subsystem$_1$ and subsystem$_2$ to represent 50% of the fault severity in the actuators. The control method validation was achieved by comparing the subsystem responses (tension deviations) without controller and LMI controller with the proposed controller. In these tests, the continuous control $u_i^{LMI}(t)$ is designed via the LMI, which described in Algorithm II, and leads to the following gain:

$$K_1 = [-521.6527 \quad 590.0355]$$
$$K_2 = [-2.5041 \quad -17.7240]$$
$$K_3 = [-10000 \quad 5663]$$

With the ISMC, the total control signal of each subsystem is:

$$u_i(t) = K_i x_i(t) - \mu_i \frac{\sigma_i(x_i, t)}{\|\sigma_i(x_i, t)\| + \mathfrak{z}_i}$$

where $i = 1, 2, 3$, $\mu_1 = \mu_2 = 2$, $\mu_3 = 3$ and $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.1$.

For healthy operations, the tension deviation of the three subsystems (x_{12}, x_{22} and x_{32}) are presented in figures 3, 4 and 5. It can be easily seen that the three tension deviations without controller are unstable. However, the LMI controller and the ISMC+LMI controller effectively regulate the tensions, stabilise the subsystem responses and provide similar performances.

The simulated faulty tension deviation signals are illustrated in figures 6, 7 and 8. Again, these results confirm that the tension deviations are unstable without controller, and they are completely stable with using the LMI or the ISMC+LMI controllers. Furthermore, figure 8 also shows an advantage to the ISMC+LMI controller over the LMI controller in regulating the tension of subsystem$_3$ as well as accommodating the effect of actuator faults, which are introduced in the subsystem$_1$ and subsystem$_2$. However, Figure 7 illustrates that the LMI controller provides better subsystem$_2$ tension deviation than the ISMC+LMI controller with the same faulty actuators. From these results, the effectiveness of the proposed controller compared with the LMI controller depends on the faulty actuator location in the system.

Further tests have also been carried out with this system when the faulty actuators in the subsystem$_3$. The results are similar to those presented here but have been omitted from this paper because of limitations of space.

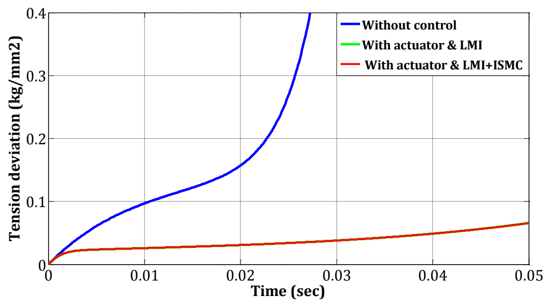

Figure 3. Tension deviation $x_{12}(t)$ of subsystem$_1$ with healthy actuator s

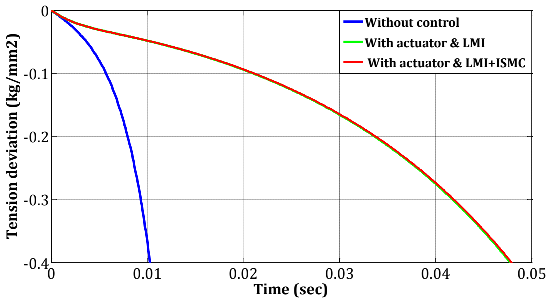

Figure 4. Tension deviation $x_{22}(t)$ of subsystem$_2$ with healthy actuators

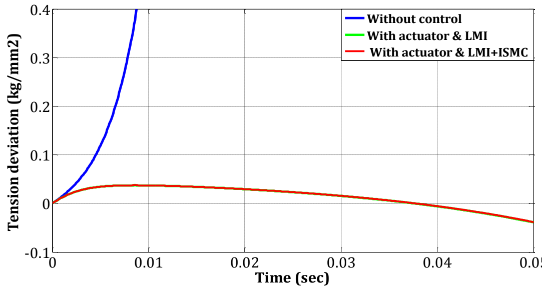

Figure 5. Tension deviation $x_{32}(t)$ of subsystem$_3$ with healthy actuators

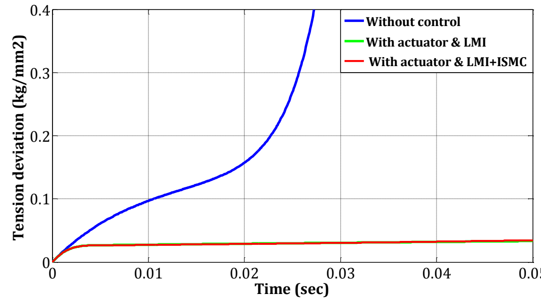

Figure 6. Tension deviation $x_{12}(t)$ of subsystem$_1$ with faulty actuators in subsystem$_1$ and subsystem$_2$

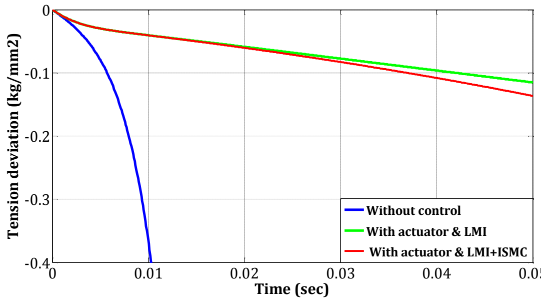

Figure 7. Tension deviation $x_{22}(t)$ of subsystem$_2$ with faulty actuators in subsystem$_1$ and subsystem$_2$

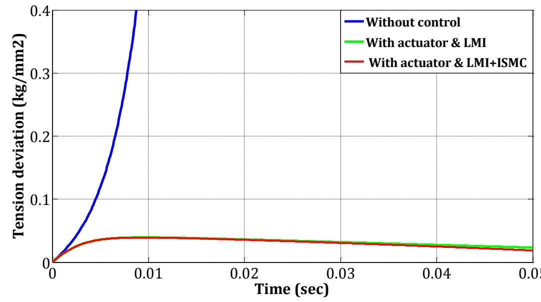

Figure 8. Tension deviation $x_{32}(t)$ of subsystem$_3$ with faulty actuators in subsystem$_1$ and subsystem$_2$

IV. CONCLUSION

This paper has demonstrated a new torque control approach for hot rolling steel mill. The proposed controller has two signals; the first is designed with the ISMC to deal with any matched components, and the second is designed via the LMI formulation to deal with any unmatched components, stabilise the system response and achieve the required performance. Also, in this work:

- The control design based on the ISMC+LMI technique has been explained.
- The control method taking into account the non-linear interaction between the inter-connected hot rolling steel systems has been presented for two control design methods.
- A set of simulation results have been obtained from MATLAB model of hot rolling system with three inter-connected systems, representing the operating condition with disturbances, healthy and faulty actuators.
- It has been shown that the proposed torque controller using the ISMC+LMI technique regulates the subsystem tension deviations, improves the responses and robustly accommodate the effects of the actuator faults and disturbances.
- The study has also clearly shown that the proposed controller not always provides better subsystem tension deviations than the LMI controller when the actuator faults are introduced, while these deviations becomes unstable without torque controller.

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