Rejection and Compensation of Periodic Disturbance in Control Systems

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Abstract— **in this paper, a brief introduction to the disturbance rejection methods is given in general and to the periodic disturbance rejection methods in particular. Therefore in the following, methods of using only feedback technique to achieve both set point tracking and periodic disturbance rejection are presented. Also, the interaction problem between the set point design demands and the periodic disturbance rejection is discussed. Then, the interaction is minimized by using a separate feed-forward controller to reject the periodic disturbances as an add-on compensator to the pre-existing set point tracking feedback controller. Furthermore, the idea of the disturbance observer is introduced in general as well as the adaptive periodic disturbance cancelation method.**

Index Terms: **periodic disturbance, feedback rejection, feed-forward compensation, periodic disturbance observers.**

I. INTRODUCTION

Undesirable oscillations are in particular assumed to be periodic disturbances, which could be generated by external sources, called forced oscillations, or internally by linear system dynamics. If one or more of the system modes are excited, this could lead to a constant oscillation (pure sinusoidal oscillation when a system has a pair of complex conjugate imaginary poles or more at the imaginary-axis in the s-plane), or a transient oscillation when the system has a positive or a negative damping coefficient.

Moreover, sustained oscillations could also take place because of nonlinear system dynamics [1]. For example, limit cycle oscillations induced from hysteresis elements, or oscillations generated because of state dependent parameters like in rotational drive-load (source-drain) systems, which could cause the rotor angular position or velocity to oscillate, and this is mainly because of angle dependent (periodic) drive or load parameters. Moreover, these types of oscillations can also be called or classified as self-excited oscillations.

In practice, such oscillations could actually be induced in the angular velocity servo control drive-load systems either from the drive side and/or from the load side. The drive side systems are, for example, recip–rocating engines, where according to their construction and working principles, they generate angle dependent pulses of torque that cause oscillation in the rotational velocity. Therefore, a lot of work has been done in order to reduce these oscillations, for example, Zaremba, Burkov and Stuntz have developed a control algorithm to reduce the oscillations of the engine idle speed [2], Gusev, Johnson and Miller have developed an active flywheel algorithm to reduce the engine speed oscillation [3] and Njeh, Cauet and Coirault have developed a new control strategy to reduce the torque ripples of the combustion engine in hybrid electric vehicles [4]. Moreover, electrical drives also have torque ripples that need to be reduced, for example, in induction motor [5, 6], as well as the cogging torque in permanent magnet synchronous-motors [7-13], etc.

On the other hand, the angle dependent load machines can also be the main source of oscillations in drive-load systems. For example, load machines with reciprocating motion like crankshaft or camshaft machines are reciprocating air compressor machines, rectilinear or reciprocating saw machines, weaving machines [14], etc.; also machines with undesirable eccentricity like disc drive systems [15, 16]; or in noncircular roll machines [17-19]. Therefore, it is very important for the performance of the drive-load system to prevent (reject) the oscillation generated at the drive side to go to the load side, or the oscillation generated at some load to affect the other coupled loads, especially, when drive-load system has more than one load linked together.

In the next, feedback disturbance rejection is briefly discussed, and then in section III, the periodic disturbance rejection filter is introduced. In section IV, the feed-forward disturbance compensation is introduced in general, then, in sections V and VI, by using disturbance observers, as well as the adaptive periodic disturbance cancellation in VII. Finally, some comments and conclusions are given in section VIII.

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II. FEEDBACK DISTURBANCE REJECTION

Set point tracking and disturbance rejection are the main objectives of the classical and the modern control system engineering, where both feedback as well as feed-forward can be used to achieve the design objectives of the set point tracking and the disturbance rejection. But, using feedback control only to achieve these objectives could lead in some circumstances to an interaction between the design demands, for example, the multi-objective design problem described in [20], subsection 2.3.3, which results in a compromising solution between the set point tracking demands and disturbance rejection. Therefore, the problem must be separated or separate solutions should be found to solve this controversy. However, in this section, the disturbance is tried to be rejected by using only the feedback control with set point tracking as a primary goal to achieve. Now, the feedback control system structure, pictured in the following Fig. 1, is assumed. The disturbed process is put under feedback control.

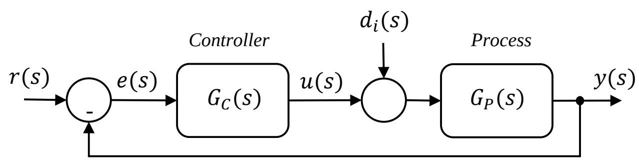

Figure. 1: FeedBack Control of Disturbed Process.

For the system shown in Fig. 1, the closed loop frequency response of the set point and the disturbance are given below respectively

$$\frac{y(s)}{r(s)} = \frac{G_P(s)G_C(s)}{1 + G_P(s)G_C(s)}, \tag{1}$$

$$\frac{y(s)}{d_i(s)} = \frac{G_P(s)}{1 + G_P(s)G_C(s)}. \tag{2}$$

From the equations (1) and (2), it can be seen that the condition for good set point tracking, which is a primary design objective here, as

$$G_P(s)G_C(s) \gg 1, \tag{3}$$

to be valid in the desired system bandwidth, which yields

$$\frac{y(s)}{r(s)} \approx 1\,; \qquad \frac{y(s)}{d_i(s)} \approx \frac{1}{G_C(s)}. \tag{4}$$

But for a perfect disturbance rejection, especially outside the systems desired bandwidth, anther more condition is needed, which is

$$G_C(s) \approx \infty, \tag{5}$$

so that the disturbance response becomes

$$\frac{y(s)}{d_i(s)} \approx 0. \tag{6}$$

However, for dynamic systems, these conditions can only be hold for specific frequency spectrum (bandwidth), so that from the disturbance rejection perspective, if the disturbance frequency still lies outside the bandwidth then it will get a poor rejection.

III. PERIODIC DISTURBANCE REJECTION FILTER

So far, the best condition for a perfect disturbance rejection is checked in the frequency domain. The second condition (of course after the validation of the first one) actually reveals how to be done. For example, if the disturbance is just a constant in the time domain, which is represented in the frequency domain by a component at frequency zero. This means that, an integral action (pole at frequency zero) must be added to the controller in order to guarantee the second condition to reject the effect of this disturbance on the system output. Therefore, for a periodic disturbance, complex conjugate poles at the disturbance frequency are needed to be added to the controller in order to keep the second condition valid for perfect disturbance rejection, which actually makes a notch in the closed loop frequency response at the disturbance frequency. This Periodic Disturbance Rejection Filter (PDRF), also "inverse" notch filter, is according to the internal model principle [21, 22]. However, most of the use of the notch filters in control applications is to shape the system frequency response of the feedback loop. They are usually implemented in a feedback control systems to suppress the resonance modes of flexible structure characteristics, which are actually the causes of the oscillations (see, for example, [23, 24]).

But now, a PDRF is considered to reject the periodic disturbance that come from an external source. The PDRF can simply be defined, for instance, by an under damped second order transfer function as

$$G_N(s) = \frac{K_d\omega_d^2}{s^2 + 2\xi_d\omega_d s + \omega_d^2}\ . \tag{7}$$

This part is simply added to the original feedback controller as shown in Fig. 2. For example, if the controller originally has the proportional, integral and the derivative actions then this part is assumed to be as an extension to the integral part to counteract the corres-ponding periodic disturbance. Fig. 3 shows the poles of the PDRF when disturbance frequency is at γ [rad/s] and its damping ratio ξ_d is equal to zero. Furthermore, Fig. 2 shows two variations of adding the PDRF to a preexisting feedback controller. The resulted closed loop transfer function will be for the variant A as

$$\begin{aligned} y_A(s) = {} & \frac{[G_C(s) - G_N(s)]G_P(s)}{1 + [G_C(s) - G_N(s)]G_P(s)}\, r(s) \\ & + \frac{G_P(s)}{1 + [G_C(s) - G_N(s)]G_P(s)}\, d_i(s), \end{aligned} \tag{8}$$

and for the variant B as

$$y_B(s) = \frac{G_C(s)G_P(s)}{1+[G_C(s)-G_N(s)]G_P(s)}r(s) + \frac{G_P(s)}{1+[G_C(s)-G_N(s)]G_P(s)}d_i(s). \tag{9}$$

Obviously, there is no difference between the variants A and B in the disturbance rejection curves. But there is a difference between the set point response curves. Variant A tries to give a large open loop gain (or even infinite for perfect disturbance rejection) at the disturbance frequency which leads to that the controller amplifies this frequency in the frequency response of the set point. This means, it gives very high open loop gain at this frequency for the set point. Therefore, this variant will be the choice to reject a periodic disturbance when its frequency is inside the system bandwidth.

Alternatively, variant B suppresses both the set point and the periodic disturbance by creating a notch in the closed loop set point and the disturbance frequency responses at the periodic disturbance frequency. This is needed when the disturbance frequency is outside the (demanded) system bandwidth. Fig. 4 and Fig. 6 show the closed loop frequency and time responses for the set point and the disturbance, when the disturbance frequency is inside the system bandwidth, while Fig. 5 and Fig. 7 show the closed loop frequency and time response, when the disturbance frequency is outside the system bandwidth.

Generally, for multi-harmonic disturbances, a number of PDRF can be designed and used to reject every single harmonic distinctively or one PDRF for a band of disturbance frequencies. For the case of infinite harmonics a repetitive controller [25-27] can be used to reject them.

In conclusion, the perfect condition for disturbance rejection will add some extra lag to the open loop path. Regrettably, this will deteriorate the closed loop set point tracking characteristics in terms of dynamics and stability. It will be even worse if there are multi-harmonics. This will make the design more complex. And at the end, there will be only a compromising solution between system (relative) stability and disturbance rejection design demands.

The design of the PDRF, added to a closed loop negative feedback controller, can actually be carried out by all classical as well as modern control design techniques. For example, if the PDRF is added to a PID controller, then the design could simply be done by tuning the PID as well as the PDRF parameters to get the desired set point tracking and periodic disturbance rejection demands. But this is not going to be an easy task for complex systems.

Alternatively, the design can be done using classical control design techniques, e.g. in frequency domain to get the corresponding phase and gain margins for a specific relative stability, or by using the pole placement technique, to get the desired stable closed loop poles. Moreover, the design can also be carried out by using modern control design techniques, e.g. using state feedback pole placement or optimal control, and furthermore alternatives are the design methods based on the robust control theory to achieve a robust controller.

Also, Iterative Learning Control (ILC) can be used to reject periodic disturbances, although it is originally designed to optimize the repeated (set-point trajectory) task tracking of robots. However, the ILC can be used to iteratively learn (estimate or adapt) a proper (feed-forward) control signal to reject a periodic disturbance given in the repeated task period, which is at the end acts as repetitive control algorithm, for example, see [2, 28].

Moreover, the ILC control learns (estimates or adapts) the control signal from the past iteration to optimize a repetitive task, rather than estimating the controller parameters as in the case of adaptive control, for more general information about ILC refer, for example, to [29, 30].

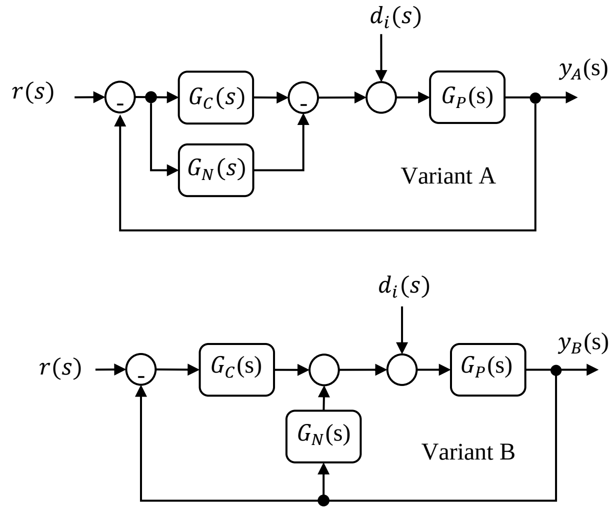

Figure. 2: Addition of PDRF in Two Variations A and B.

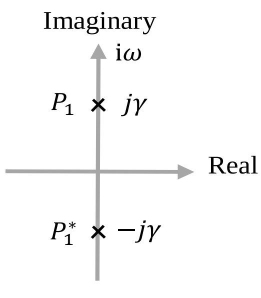

Figure. 3: The PDRF Poles in s-Plane for $\omega_d = \gamma$ and $\xi_d = 0$.

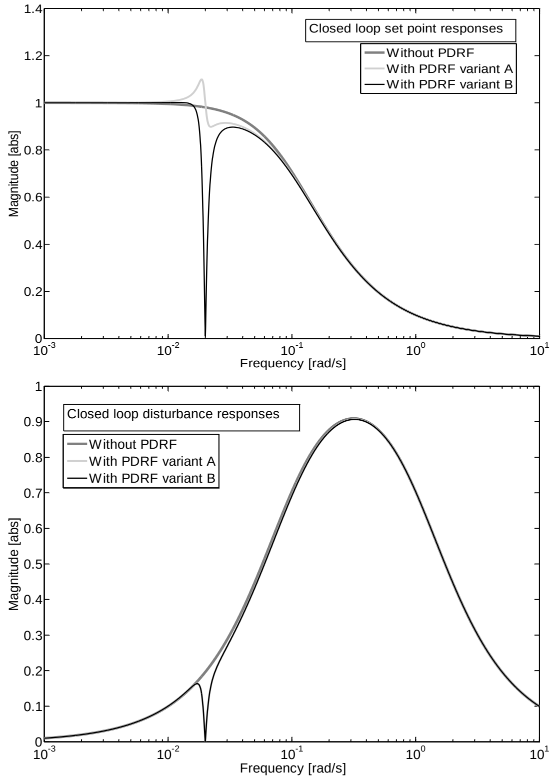

Figure. 4: Disturbance Frequency Inside the (Demanded) System Bandwidth.

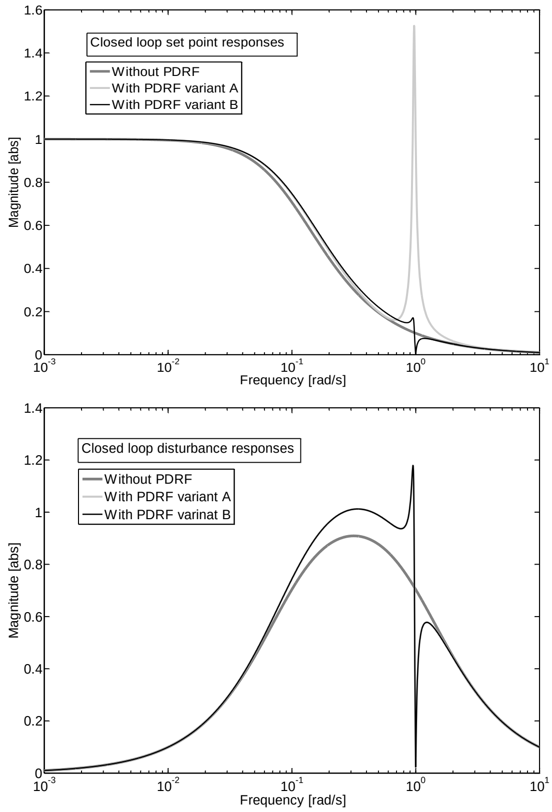

Figure. 5: Disturbance Frequency Outside (Demanded) System BandWidth.

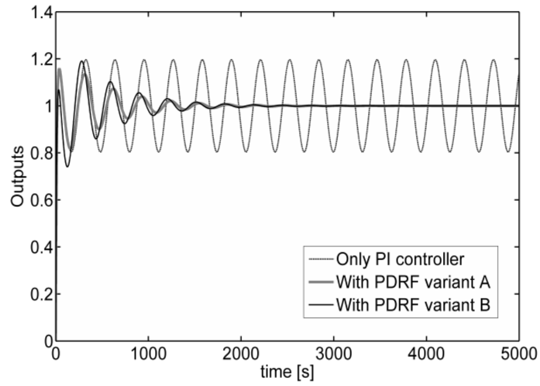

Figure. 6: Disturbance Frequency Inside (Demanded) System BandWidth.

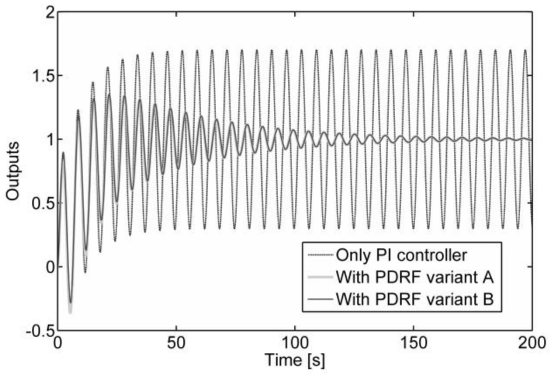

Figure. 7: Disturbance Frequency Outside (Demanded) System BandWidth.

IV. FEED-FORWARD DISTURBANCE COMPENSATION

To achieve feed-forward compensation, both the disturbance signal and the model characteristics of the system should be known, this can be explained by the following. A system, as shown in Fig. 8.A, is assumed, which is described by functional (operator)

$$y(t) = f(u(t), d(t)), \qquad (10)$$

where y(t) is the output, u(t) is the (manipulated) input and d(t) is the disturbance input of the system. Furthermore, the disturbance is assumed to be measurable and its effect on the system output is needed to be cancelled by using the manipulated input. A further assumption is to be made that the input signal and the disturbance signal have an independent action on the system output. This means, that the output functional is separable, see Fig. 8.B1, and can be separated into independent functionals as

$$y(t) = f_u(u(t)) + f_d(d(t)). \qquad (11)$$

Now, if the disturbance action is to be compensated by the input signal, this makes its effect on the output equal to zero, see Fig. 8.B2. From this, the input compensation signal (feed-forward control law) can be calculated by

$$u(t) = -f_u^{-1}(f_d(d(t))), \qquad (12)$$

where $f_u^{-1}(u(t))$ is the inverse of the input functional. Now, if the disturbance action on the output is to be compensated by the input, then the disturbance signal must be available either by direct measurement, observation (if it is not directly measured), estimation (measurements are stochastic) or prediction (if the needed value is in future, especially for systems with large time delay). In addition, the system input functional, its inverse and the disturbance functional should be also available (known). Furthermore, if the input and the disturbance functionals can be represented by linear transfer functions, as shown in Fig. 8.C1, then

$$y(s) = G_u(s)u(s) + G_d(s)d(s). \qquad (13)$$

So again, the condition for disturbance compensation by the input, as shown in Fig. 8.B2, is given by

$$u(s) = -\frac{G_d(s)}{G_u(s)}d(s). \qquad (14)$$

Apart from the lately discussed conditions, the input transfer function has to have minimum phase (stable) zeros and the ratio of the disturbance transfer function divided by the input transfer function should be causal (denominator degree is higher than the degree of the numerator). Otherwise, the feed-forward control law is unrealizable. Notwithstanding this, an exception can though be made, if the disturbance is a periodic signal, and the feed-forward law is non-causal. Therefore, it will be a matter of finding the right amplitude and phase shift to compensate (cancel out) the respective periodic signal. This will be more discussed in section VII.

Fig. 8.D1 shows the output disturbance format and Fig. 8.E1 shows the input disturbance format. The transformation of the output disturbance into input disturbance is given by

$$d_i(s) = \frac{1}{G_u(s)}d_o(s)\,, \qquad (15)$$

where d_o is the output disturbance ($d_o(s) = G_d(s)d(s)$), while the transformation of the input disturbance to output disturbance is given by

$$d_o(s) = G_u(s)d_i(s), \qquad (16)$$

It is clear from the transformation equations that the input to output disturbance transformation is causal, but the output to input disturbance is not, since the reciprocal of causal dynamics yields a non-causal system. For real application, an exception can be made only for periodic disturbances, since the periodic signals are completely predictable in future when their amplitude, phase shift and frequency are constants.

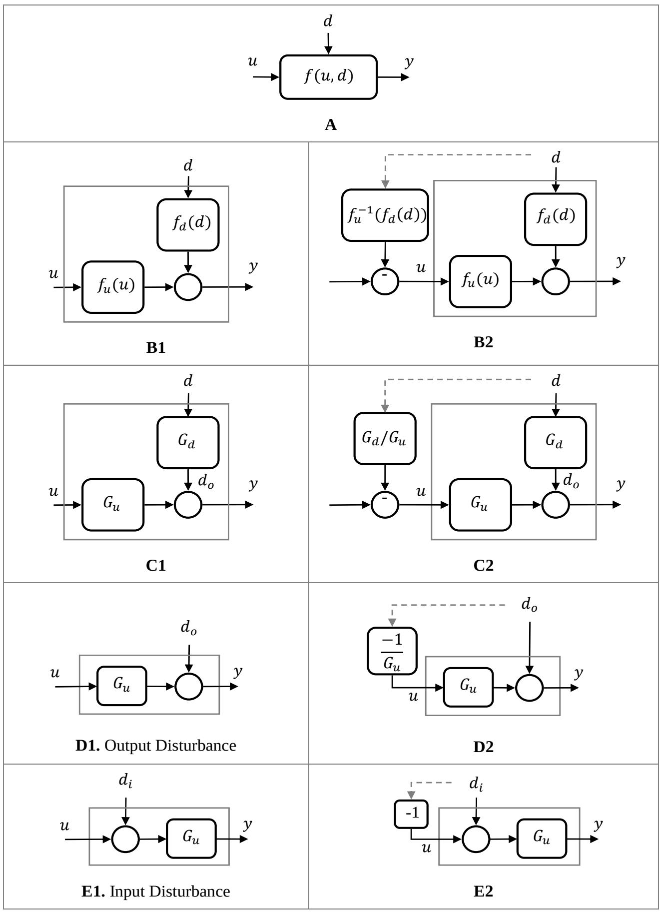

Figure. 8: Disturbances and their Feed-Forward Control.

Now, assuming that the structures given in Fig. 8.A, B1, C1 and D1 can be transformed into the form of the direct input disturbance, see Fig. 8.E1, by having this signal as a direct measurement, observation, estimation or prediction, then it can be used directly without extra computation, or simply the feed-forward control law becomes as

$$u(t) = -d_i(t)\,. \qquad (17)$$

So at the end, it is better to get the input disturbance signal even when the real system has another disturbance signal, e.g. output disturbance signal, so that to get the simplest feed-forward control law, as shown in Fig. 8.E2.

Regrettably, in practice, most of the cases are either the direct measurement, by using extra sensor(s), is impossible, or it is commercially too expensive to realize. Therefore, indirect methods are used, for example, using a disturbance observer (estimator, predictor) to construct the disturbance signal by monitoring the system input and the disturbance effect on the output. However, the implementation of this type of disturbance observer to estimate the disturbance and at the same time to cancel its effect on the output, transforms the principle idea of feed-forward control back into feedback control, but one exception could be made when the process model is perfect. Therefore, this type of feed-forward control is sometimes alternatively called estimated feedback control, pseudo or virtual feed-forward control [20].

V. INTRODUCTION TO DISTURBANCE OBSERVERS

In this section, some simple disturbance observers are briefly introduced and discussed, first using transfer function filter formats and then later in state space.

A. Transfer Function Based Disturbance Observer

The problem here is to compute the output disturbance of a process from its input and output measurements, where the output measurements are also assumed to be corrupted with a measurement error (noise), as shown in Fig. 9.

$$y_m(t) = y_u(t) + d_o(t) + e_m(t);$$
$$y_m(s) = G_P(s)u(s) + d_o(s) + e_m(s). \quad (18)$$

Now, if the disturbance estimate is needed and the process dynamics can be represented or approximated by some mathematical model, so that the response of the process dynamics to the input can be estimated by

$$\hat{y}_u(s) = \hat{G}_P(s)u(s). \quad (19)$$

Consequently, the output disturbance can be computed (estimated) by

$$\hat{d}_o(t) = y_m(t) - \hat{y}_u(t). \quad (20)$$

Since the process dynamics cannot in reality be structured in a model without any error, or in other words, there is actually an error, because of a model structure that cannot take in consideration all modes or nonlinear characteristics of the process. Therefore, this modeling error is defined as a disturbance added to the estimated output of the process dynamics

$$y_u(t) = \hat{y}_u(t) + d_{ME}(t). \quad (21)$$

So, the disturbance computation will be

$$\hat{d}_o(t) = \quad y_u(t) + d_o(t) + e_m(t)$$
$$-(y_u(t) - d_{ME}(t)), \quad (22)$$

which gives

$$\hat{d}_o(t) = d_o(t) + e_m(t) + d_{ME}(t). \quad (23)$$

Thus at the end, the computation of the disturbance is corrupted with the measurement and the modeling error of the system. Moreover, the last disturbance observer computes the estimate of the output disturbance, which is not directly applicable if its effect on the output is wanted to be compensated. Since it must be first converted to an input disturbance and that needs the inverse of the estimated process dynamics, as in equation (24) and Fig. 9 show.

$$\hat{d}_i(s) = \left(\hat{G}_P(s)\right)^{-1} \hat{d}_o(s), \quad (24)$$

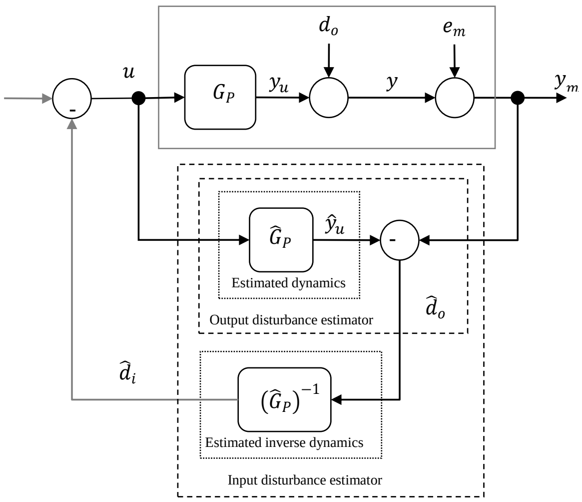

Figure. 9: Output-Input Disturbance Observer.

provided that the (estimated) process has no non-minimum phase (unstable) zeros and its inverse is a causal system. This way of computing the input disturbance is indirect and susceptible to a lot of computation errors, since the estimated process dynamics and their inverse are needed in the computation. For a better performance, an input disturbance observer is introduced that uses only the estimate of the process dynamics, plus a feedback correction is applied by amplifying the observer error between the real (measured) and the estimated output as shown in the Fig. 10. So, the estimated input disturbance is given as

$$\hat{d}_i(s) = \frac{G_c(s)G_P(s)}{1 + G_c(s)\hat{G}_P(s)} d_i(s). \quad (25)$$

For the case that $G_c(s)\hat{G}_P(s) \gg 1$, the estimated disturbance becomes

$$\hat{d}_i(s) = \frac{G_P(s)}{\hat{G}_P(s)} d_i(s), \quad (26)$$

and here, the input disturbance is estimated without the need to compute the inverse of the estimated dynamics with the condition that $\hat{G}_P(s) = G_P(s)$. The estimated input disturbance will be exactly equal to the real input disturbance. Of course, the condition will hold only for a specific bandwidth determined by the closed loop poles.

Moreover, the design of the G_c depends on the type of the disturbance either be a real external independent disturbance acting on the system or an internal disturbances coming from unknown or un-modeled dynamics. For more practical information refer to [31], and for the disturbance observer Q-Filter type refer to [32-34]. So generally, when the disturbance is simply a constant bias or its time or frequency behavior is unknown. Then, it can be generally modeled as (variable) constant when the observer bandwidth or their poles are (as a rule of thumb) at least ten times faster than

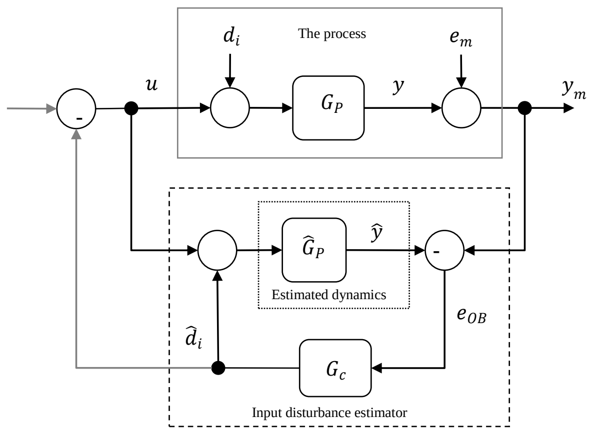

Figure. 10: Computing the Input Disturbance without Using the Inverse of Estimated Dynamics.

the disturbance bandwidth. This leads of course to high gain design that its application could be limited by the noise ratio in the system. Otherwise, it would be better to be modeled within G_c according to the IMP, this could achieve better observation with relatively low observer gain. The idea is also presented using state space models in section VI.

B. Model Following as Disturbance Canceller

Now, instead of observing the (input) disturbance acting on a process and then using its estimated signal to compensate the disturbance effect on the output, alternatively, the process can be forced to follow specified dynamics given as a (linear/nonlinear) model. By doing so, the output response becomes as a projection of the specified dynamics, even when the process has external disturbances, as long as these disturbances and model mismatches are in the working bandwidth of the closed loop system.

This can be done, for example, by considering the (inexact) model following case, particularly when the process inverse dynamics are not available or unknown, therefore from Fig. 11, when the block $(\hat{G}_P(s)/G_P(s))$ is substituted by or put equal to one, then the system output becomes

$$y_m(s) = \frac{G_P(s)\left(1 + G_c(s)\hat{G}_P(s)\right)}{1 + G_c(s)G_P(s)} r(s) + \frac{d_o(s)}{1 + G_c(s)G_P(s)} + \frac{e_m(s)}{1 + G_c(s)G_P(s)}. \quad (27)$$

From equation (3.27), if $\hat{G}_P(s) = G_P(s)$ then the closed loop response to the new input $r(s)$ becomes $G_{CL}(s) = G_P(s)$, and for the case, when $\hat{G}_P(s) \neq G_P(s)$, $G_c(s)G_P(s) \gg 1$ and $G_c(s)\hat{G}_P(s) \gg 1$ the closed loop response becomes $G_{CL}(s) \approx \hat{G}_P(s)$.

This means that the closed loop system follows the desired dynamics $\hat{G}_P(s)$ in perspective of the new input $r(s)$. This concept is a complementary concept of the disturbance observer, where the disturbance cancelling is done by using the estimated dynamics and the disturbances, while (inexact) model following is done by giving the desired system dynamics.

Moreover, the system suppresses the effect of the disturbance and the measurement error in the specified system bandwidth where the condition $(G_c(s)G_P(s) \gg 1)$ holds, for more details about model following control refer, for example, to [35].

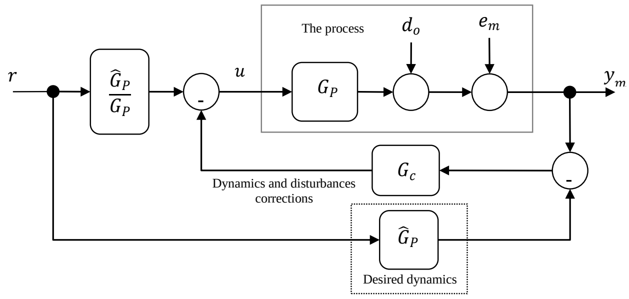

Figure. 11: Model Following Control Concept.

VI. STATE AND DISTURBANCE OBSERVER IN STATE SPACE

The modern state observers can be traced back to 1960, where the idea of the stochastic state observer [36] was introduced and in 1961 developed as Kalman-Bucy-Filter [37]. Later in 1964, the deterministic version was reintroduced by Luenberger [38]. For more information about the state and disturbance observers refer, for example, to [39].

A. Extended Sinusoidal Disturbance Observer

The extended disturbance can be generated by using a sinusoidal signal generator in state space format with a constant angular frequency (see, e.g. [40]) as

$$x_{d1}(t) = sin(\omega t), \quad (28)$$

$$x_{d2}(t) = \dot{x}_{d1}(t) = \omega\, cos(\omega t), \quad (29)$$

$$\dot{x}_{d2}(t) = -\omega^2\, sin(\omega t) = -\omega^2 x_{d1}(t). \quad (30)$$

The sinusoidal generator system can be put in a state space model as

$$\begin{bmatrix}\dot{x}_{d1}(t)\\ \dot{x}_{d2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega^2 & 0\end{bmatrix}\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix};$$
$$y_d(t) = [1 \quad 0]\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix}. \quad (31)$$

Alternatively, the states can also be defined as

$$x_{d1}(t) = sin(\omega t);\ x_{d2}(t) = cos(\omega t), \quad (32)$$

$$\dot{x}_{d1}(t) = \omega\, cos(\omega t) = \omega x_{d2}(t), \quad (33)$$

$$\dot{x}_{d2}(t) = -\omega\, sin(\omega t) = -\omega x_{d1}(t). \quad (34)$$

Then, the state space sinusoidal generator system can alternatively be put in the form

$$\begin{bmatrix}\dot{x}_{d1}(t)\\ \dot{x}_{d2}(t)\end{bmatrix} = \begin{bmatrix}0 & \omega\\ -\omega & 0\end{bmatrix}\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix};$$
$$y_d(t) = [1 \quad 0]\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix}, \quad (35)$$

or generally as variable frequency derived from a time variant angular position $\varphi(t)$

$$x_{d1}(t) = sin\big(\varphi(t)\big);\ x_{d2}(t) = cos\big(\varphi(t)\big), \quad (36)$$

$$\dot{x}_{d1}(t) = \dot{\varphi}(t)\, cos\big(\varphi(t)\big) = \dot{\varphi}(t) x_{d2}(t), \quad (37)$$

$$\dot{x}_{d2}(t) = -\dot{\varphi}(t)\, sin\big(\varphi(t)\big) = -\dot{\varphi}(t) x_{d1}(t). \quad (38)$$

Therefore, the state space model becomes as

$$\begin{bmatrix}\dot{x}_{d1}(t)\\ \dot{x}_{d2}(t)\end{bmatrix} = \begin{bmatrix}0 & \dot{\varphi}(t)\\ -\dot{\varphi}(t) & 0\end{bmatrix}\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix};$$
$$y_d(t) = [1 \quad 0]\begin{bmatrix}x_{d1}(t)\\ x_{d2}(t)\end{bmatrix}. \quad (39)$$

There are a lot of periodic disturbance compensation methods based on disturbance observers, for example, by designing a set of linear time-invariant observers for a set of disturbance frequencies in some operating regions, that results in gain scheduled control, for example, Bohn, Cortabarria, Härtel and Kowalczyk have suggested that the observer design can be done by using pole placement techniques or by designing an optimal stationary Kalman filter [41].

Moreover, robust control LPV-techniques can also be used to design the observer and controller vectors, for example, Du and Shi as well as Du, Zhang, Lu and Shi have developed H∞ and LPV methods respectively for active vibration control applications [42, 43]. Furthermore, Ballesteros and Bohn have developed and applied the robust control LPV design algorithms in the discrete-time format [44, 45]. In addition, Kinney and de Callafon have developed a study of rapidly varying frequencies regulation guarantee [46]. For extra discussion about the so called "waterbed effect" or "spillover" that accompanies these algorithms, see for example, [41] and [47].

B. State and Disturbance Observer as Model Follower

The state observer can also be interpreted to work as a model follower, for instance, if the desired system dynamics are given in the state observer model, then the disturbance state can be used to cancel the external disturbances and internal disturbances in terms of dynamic difference between the desired model and the real one. But these again need high loop gain, or it will only work in the observer bandwidth represented by the observer poles, which have been set by the observer gain vector.

VII. ADAPTIVE PERIODIC DISTURBANCE CANCELATION

Another way of cancelling the periodic disturbance effect on the system output is by using a direct estimation and cancelation methods of the input periodic disturbance. These methods have been mostly developed by the signal processing community for active noise (sound) and vibration cancelation (control) applications. For example, consider the system presented in Fig. 12 below, where the process is disturbed by a harmonic input disturbance with a known frequency (ω_d). Therefore, a simple solution is to generate a harmonic (sinusoidal) signal as the following equation

$$u(t) = -\hat{d}_i(t) = \alpha\, sin(\omega_d t) + \beta\, cos(\omega_d t), \quad (40)$$

or alternatively as

$$u(t) = -\hat{d}_i(t) = a_d\, sin(\omega_d t + \emptyset_d), \quad (41)$$

where

$$a_d = \sqrt{\alpha^2 + \beta^2}\ ; \quad \emptyset_d = tan^{-1}\left(\frac{\beta}{\alpha}\right), \quad (42)$$

and to tune its parameters, the amplitudes α and β, or magnitude a_d and phase shift $\emptyset_d$, until the effect of the harmonic disturbance is completely (vanished) canceled out [48]. The parameter adaptation could be practically done manually as Conover (1956) demonstrated an active noise cancellation system for transformer noise [49], or by using adaptive algorithms, for example, the least mean squares algorithm and its variants [49-53]. These algorithms are usually known as adaptive feed-forward in signal processing literature, although they are "truly and purely a feedback control law" [22]. Moreover, Bodson has shown that these adaptive feed-forward controllers can be equivalent under certain conditions to the internal model principle linear controllers [54].

However, the advantage of using the LTI-IMP periodic disturbance observer is that the closed loop stability can be computed and proven, provided that an accurate mathematical model for the targeted process is available at least in the operating bandwidth. But, on the other hand, for this adaptive feed-forward controller, the closed loop stability analysis depends on the adaptation algorithm used to tune its parameters which has usually a time-varying nonlinear characteristics, this makes the stability analysis difficult to prove in general.

Nevertheless, an advantage of this adaptive feed-forward control, when the adaptive algorithm is deactivated after the convergence of its parameters to the optimal ones, especially when the disturbance parameters are not time varying, so that the estimated disturbance signal compensates or cancels out the periodic disturbance effect on the system output. Then, at this situation the pseudo feed-forward controller becomes a true feed-forward controller, and therefore it will not affect the system closed loop dynamics anymore, where they can then be freely designed to fulfill the stability and set point design demands separately.

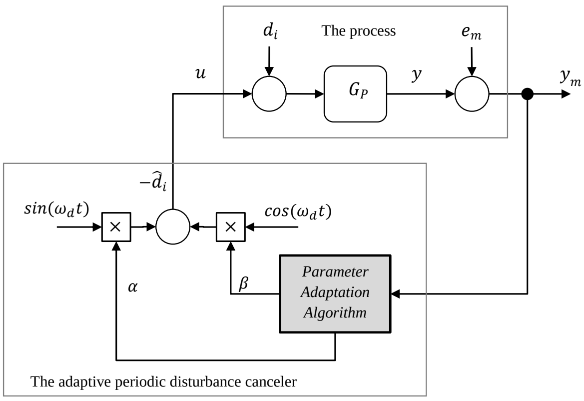

Figure. 12: Adaptive Periodic Disturbance Canceller.

VIII. COMMENTS AND CONCLUSIONS

As presented in this paper, methods of periodic disturbance rejection by using feedback control only as well as methods of periodic disturbance compensation by using add-on feed-forward control are briefly introduced, where their similarities, advantages and disadvantages between the algorithms are pointed out and discussed.

As has been shown, the periodic disturbance can be rejected to some extent by using only a feedback controller, for example, a PID controller. But for most of the design circumstances the periodic disturbance is not perfectly rejected except if the model of the periodic disturbance is belt in the controller, which is according the internal model principle, for example, by implementing the periodical disturbance rejection filter. However, this is going to lead to that the controller design will be further more complicated, and even worse sometimes, there will be an interaction between the set point tracking and periodic disturbance rejection demands.

In general, for the case of infinite harmonics (periodic) disturbances a repetitive controller can be used to reject them, also the iterative learning control can be applied to reject the periodic disturbances, although it is originally designed to target the repeated set point trajectory tracking.

Therefore, methods of the feed-forward algorithms are explored, where the feed-forward is implemented by using a disturbance observer to estimate the disturbance. Where, the disturbance observer is constructed either as transfer function model based or state space model based. However, by using the estimated disturbance from these types of observers that use the measured output in the disturbance estimation, this makes the system works as pure feedback system, though an exception can be made, when the model is perfect, but this case is seldom in practice. Therefore, the problem of set point tracking and the periodic disturbance rejection will no longer be separate. Also, the interpretation of the model follower as disturbance canceller is given, both for the transfer function model based and the state space model based algorithms.

Consequently, the methods of adaptive periodic disturbance cancelation, intensively implemented in active noise and vibration control applications, are reviewed, where, they turned out to be working the same as a feedback controller with more complication because of the adaptation algorithms, but with still minimum advantage that can be a great benefit in practice, particularly, when the parameters converge to the optimal ones and the adaptation is switched off, then the feed-forward controller becomes a true feed-forward controller, this will separate the set point tracking design demands and the periodic disturbance rejection.

Moreover, this work has been done with the aim of developing and applying these algorithms to suppress the periodic disturbances that take place in drive-load (source-drain) systems in general, and in particular for example, to suppress the vibrations generated by the drive (rotational reciprocating engine) in automobile flexible chasse as well as damping the vibration side effect (noise) in the passenger compartment to increase the comfort factor in the car.

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