Length Estimation of Single-mode Fiber Links under Group Velocity Dispersion GVD Effect using 1 dB Power Penalty Toleration

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Abstract— Intermodal dispersion in multimode fibers leads to considerable broadening of optical pulses during transmission. Although intermodal dispersion can be overcome by using single-mode fibers, pulse broadening does not disappear altogether. This is due to another phenomenon called group velocity dispersion. The main goal of this paper is to estimate the maximum optical transmission length that can be achieved under the effect of group velocity dispersion GVD when single-mode fiber is used. Optical sources with large spectral width and small spectral width were assumed in this study. Bit rates of 622 Mbps, 1 Gbps, 2.5 Gbps, and 10 Gbps were assumed in the study as well, as they are the most commonly used in the optical access systems. The study was performed using Matlab software (The Mathworks, Inc., Natick, MA, USA). Although a significant improvement were observed when using an optical source with small spectral width in which the GVD noise floor is reached at much longer fiber, a dramatic reduction in the transmission distance is encountered when the bit rate reaches 10 Gbps, which necessitates incorporation of a dispersion compensation technique in the transmission line.

Index Terms: optical access, pulse broadening in optical fiber, group velocity dispersion GVD, inter-symbol interference ISI, power penalty.

I. INTRODUCTION

Although intermodal dispersion is not an issue in single mode fibers (SMFs) as the energy of an input pulse to an SMF is transferred by only one mode (the fundamental mode), a phenomenon referred to as chromatic dispersion, also called group velocity dispersion GVD, leads to the broadening of pulses as they propagate through an SMF [1]. This is due to the dependency of the group velocity associated with the fundamental mode on frequency. Because an optical pulse occupies a finite frequency span, its width will be broadened as it propagates through an SMF due to the GVD effect. In any optical communication system, GVD could affect the performance by:

First: Decreasing the pulse peak power, which leads to reduce the pulse energy within its allocated bit slot, and consequently reduce the associated SNR measured at the receiver circuit. Because the SNR should remain constant to maintain a desirable BER performance, an increase in the average power at the receiver is required. This increase in the average power is referred to as the power penalty and is imposed to compensate for energy reduction within the bit slot at the receiver decision circuit. The power penalty ($P_P$) is defined as the additional power required to maintain a specific bit-error rate (BER) performance and is expressed in decibels as [2]

$$P_P = 10 \log_{10} \left( \frac{\text{Power with impairment}}{\text{Power without impairment}} \right)$$

Secondly: Spreading the pulse energy out of its allocated bit slot. This makes the pulse overlaps with adjacent pulses (inter-symbol interference ISI), and thus leads to degrade the performance. To reduce the ISI effect, the receiver should be designed in such a way that makes the input signal to its decision circuit matches the transfer function of a raised cosine filter [3]. These effects can be understood more clearly by referring to Figure 1.

Figure 1. an Optical Gaussian Pulse and its Broadened Version after Transmission over a Span of Single-Mode fiber

Many researches have been conducted in the field of dispersion in optical fibers. Ali Ebadi et al all simulated and calculated the dispersion for an ideal optical fiber, and studied the total dispersion and waveguide dispersion and their relationship with power confinement [4], Yu Zhao et all proposed an analysis method for mode dispersion of optical fiber based on the approaches of the digital holographic microscopic tomography and ray tracing...
theory. While the approach of ray tracing theory was used to simulate the multimode fiber transmission and analyze the transmission characteristics of both perfect and defective optical fibers, the digital holography was used to measure the refractive index of multi-mode fiber by the refractive index of optical fiber testing system [5]. Hao-Jan Sheng et al investigated the optical dispersion phenomena in a side-polished single-mode superstructure fiber grating by applying different bending curvatures including group velocity delay, chromatic dispersion [6]. Shcherbakov et al all presented experimental results on transmission of signals in analog fiber-optic links with direct intensity modulation and direct detection of photocurrent while using dispersion compensating fiber (DCF) to compensate for the accumulated dispersion. The results show an appreciable mitigation of the depth of signal power dips was observed despite the existence of the residual total dispersion [7]. Franco et al designed a defected-core microstructured optical fiber (MOF) to compensate the residual chromatic dispersion and evaluated its feasibility using dimensional sensitivity analysis. The MOF design exhibits ultraflattened negative dispersion over S, C, L, and U wavelength bands and average dispersion of about -179 ps/nm/km with an absolute dispersion variation of 2.1 ps/nm/km from 1480 to 1675 nm (195-nm bandwidth) [8]. In this paper, the effect of group velocity dispersion GVD was studied to estimate the maximum optical transmission distance that can be achieved when single-mode fiber is used. Optical sources with large spectral width and small spectral width were assumed in this study. Bit rates of 622 Mbps, 1 Gbps, 2.5 Gbps, and 10 Gbps were assumed in the study as well, as they are the most commonly used in the optical access systems. For example, 622 Mbps for BPON, 1 Gbps for EPON, 2.5 Gbps for GPON, and 10 Gbps for 10G-EPON and XG-PON.[9][10][11][12][13][14][15].

II. GVD ANALYTICAL MODELING

In any practical optical communication system, an optical pulse does not have an exact rectangular shape. Statistically, the best approximation that might describe the optical pulse is the Gaussian representation (bell-shaped representation). This simplifies describing the signal pulse by its average (mean), mean square (variance), and standard deviation (root mean square). To estimate the power penalty imposed by the GVD effect, refer to Figure 1. The figure shows an input Gaussian pulse and its broadened version. The RMS of the given input signal $\sigma_o$ is given by $\sigma_o = \frac{T_o}{\sqrt{2}}$, where $T_o$ controls the pulse width and is related to the pulse full width at half maximum as $T_{FWHM} = 1.665T_o$. Using approximately Gaussian statistics and accounting for both the data and source spectrum widths, the variance of the output broadened pulse $\sigma^2$ can be given as [16].

$$\sigma^2 = \sigma_o^2 \left(1 + \frac{C \beta^2 L}{2\sigma_o^2}\right)^2 + \sigma_o^2 \left(1 + V_o^2 \left(\frac{\beta^2 L}{2\sigma_o^2}\right)^2\right)$$

$$\sigma_o^2 (1 + C^2 + V_o^2) \left(\frac{\beta^2 L}{4\sqrt{2}\sigma_o^2}\right)^2$$

(2)

This variance in this case is the sum of the variances of the input pulse and its associated dispersion ($\sigma^2 = \sigma_o^2 + \sigma_D^2$), where $C$ governs the frequency chirp imposed on the pulse, $\beta_2$ and $\beta_3$ represent the second- and third-order derivatives of the propagation constant $\beta$, respectively, and $V_o = 2\sigma_o\sigma_o$, where $\sigma_o$ represents the RMS of the source spectrum width [11]. While many cases can be presented by observing equation (2), the case where C-band is nominated for optical transmission was considered solely due to the low attenuation of this frequency band. Since transmission in C-band means transmission away from the zero dispersion region, the term $\beta_3$ is assumed to be zero in this case. Two different types of optical sources were assumed in the modeling process (an optical source with large spectral width and an optical source with small spectral width are considered, respectively).

A. GVD analytical modeling with large spectrum width optical source

In this case, $V_o$ in (2) is assumed to be $>> 1$, which makes the term $(1 + V_o^2) \approx V_o^2$. To simplify the analysis, we consider a chirp-less input Gaussian with $C = 0$. Considering the above yields the following:

$$\sigma^2 = \sigma_o^2 \left[1 + \left(\frac{V_o^2}{2\sigma_o^2}\right)^2\right]$$

(3)

Substituting $V_o = 2\sigma_o\sigma_o$ yields

$$\sigma^2 = \sigma_o^2 + (D L \sigma_o)^2 = \sigma_o^2 + \sigma_D^2$$

where $\beta_2\sigma_o = D\sigma_o$. $D$ is called the dispersion parameter which is measured in ps/(nm x km), and $\sigma_o$ is the RMS of the source spectrum width in wavelength unit. The RMS of the output broadened pulse is simply obtained by taking the square root of its variance and is given by $\sigma = (\sigma_o^2 + \sigma_D^2)^{1/2}$, where $\sigma_D$ is the RMS dispersion, which is related to the dispersion parameter, the fiber length, and the RMS of the source spectrum by $\sigma_D = |D| L \sigma_o$. Dividing the RMS of the output broadened pulse by the RMS of the input pulse gives a normalized estimation of the pulse broadening value. This unit-less value is called the broadening factor $B_F$ and can be derived as follows:
\[
\frac{\sigma^2}{\sigma_o^2} = \frac{\sigma}{\sigma_o}^2 = \left[\frac{(\sigma_o^2 + \sigma_D^2)}{\sigma_o^2}\right] = \left[1 + \left(\frac{\sigma_D}{\sigma_o}\right)^2\right] = \left[1 + \left(\frac{DL\sigma_D}{\sigma_o}\right)^2\right].
\]

Taking the square root of \(\frac{\sigma}{\sigma_o}\), yields
\[
B_F = \frac{\sigma}{\sigma_o} = \left[1 + \left(\frac{DL\sigma}{\sigma_o}\right)^2\right]^{\frac{1}{2}}.
\]

We relate the broadening factor \(B_F\) to the bit rate \(B_t\) by using the criterion \(\sigma B_F \leq \frac{1}{4}\), which guarantees keeping 95% of the pulse energy within its allocated bit slot and thus leads to give the maximum bit rate as \(B_t = \frac{1}{4\sigma} \) .

Considering this criterion and considering that \(\sigma_o = \frac{\sigma}{B_t}\), equation (4) becomes
\[
B_F = \left[1 + \left(\frac{DL\sigma}{\sigma_o}\right)^2\right]^{\frac{1}{2}}
\]

given that \(\sigma = \frac{1}{4B_t}\).

yielding \(B_F = \left[1 + (DL\sigma B_t B_F)^2\right]^{\frac{1}{2}}\). Solving this equation for \(B_F\) yields
\[
B_F = \frac{1}{\left[1 - (4DL\sigma B_t)^2\right]^{\frac{1}{2}}}
\]

Because the broadened optical pulse has the same energy as the input pulse, an associated decrease in the peak power due to the GVD is expected by the same factor \(B_F\), which leads to a consequent power penalty in dB to maintain a constant energy within the bit slot \(T_b\) and thus a constant SNR. The power penalty in dB due to the GVD effect is defined as the increase in average power required for maintaining a specific BER and is given by
\[
P_{\text{GVD}} = 10\log_{10}B_F = -5\log\left[1 - (4DL\sigma B_t)^2\right]
\]

B. GVD analytical modeling with small spectrum width optical source

In this case, \(V_o\) in (2) is assumed to be << 1, which makes the term \(1 + V_o^2 \approx 1\). To simplify the analysis, we consider a chirp-less input Gaussian with \(C = 0\). Considering the above yields
\[
\sigma^2 = \sigma_o^2 \left[1 + \left(\frac{\beta L}{2\sigma_o^2}\right)^2\right]
\]

From (7), \(\sigma^2 = \left[\sigma_o^2 + \sigma_o^2\left(\frac{\beta L}{2\sigma_o^2}\right)^2\right]\), which is equivalent to \(\sigma^2 = \sigma_o^2 + \sigma_D^2\). In this case, the dispersion-induced broadening depends on the initial width \(\sigma_o^2\).

It is found that \(\sigma\) is a minimum at
\[
\sigma_o = \left(\frac{\beta L}{2}\right)^{\frac{1}{2}}
\]
and is given by \(\sigma = (\frac{|\beta| L}{2})^{\frac{1}{2}}\) [16].

Using the criterion \(\sigma B_F \leq \frac{1}{4}\) and following the same procedures followed in the first case, the power penalty due to the GVD effect in this case can be given as
\[
P_{\text{GVD}} = -5\log_{10}\left[1 - (2B_t\sqrt{|\beta L|})^2\right]
\]

III. RESULTS AND DISCUSSIONS

This section is divided into two parts. In the first part, the maximum optical transmission length is estimated under the effect of GVD in case of using an optical source with large spectrum width; whereas in the second part, the maximum optical transmission length is estimated under the effect of GVD in case of using an optical source with small spectrum width. A maximum of 1 dB was suggested as allowable power penalty because it leads to small percent of increase in the power (i.e. 25 %) compared for example with 2 dB or 3 dB power penalties which would lead to 60 % or 100 % of increase in power, respectively. Such small increase in the power would ensure non occurrence of non-linear effects in the transmission link.

A. Length Estimation when Large Spectrum Width Optical Source is Used

In this case, the maximum transmission length is evaluated by considering equation (6).

Figure 2 shows the graphical representation of (6). It represents the power penalty vs. fiber length at different bit rates (622 Mbps, 1 Gbps, and 2.5 Gbps). It is clear that the power penalty increases in each case as the fiber length increases until it reaches a point where it tends to infinity (GVD noise floor).
The GVD noise floor occurs most rapidly at the highest bit rate (2.5 Gbps), which reduces the fiber length to the shortest (1.9 km). In general, the fiber lengths that can be used if 1 dB penalty is allowed are 7.64 km at 622 Mbps, 4.75 km at 1 Gbps, and 1.9 km at 2.5 Gbps, respectively. These findings invalidate the use of large spectrum-width optical sources in the optical transmitter because a minimum length of 20 km is specified for optical access.

B. Length estimation when small spectrum width optical source is used

In this case, the maximum transmission length is evaluated by considering equation (8). Figures 3 (a), (b), and (c) show the graphical representations of (8). They represent the power penalty vs. fiber length at different bit rates (622 Mbps, 1 Gbps, and 2.5 Gbps). It is obviously seen that a significant improvement is achieved in which the GVD noise floor is reached at much longer fibers compared with the first case. The fiber lengths that can be used in this case if 1 dB penalty is allowed are 2988.6 km at 622 Mbps, 1156.25 km at 1 Gbps, and 185 km at 2.5 Gbps, respectively. These findings strongly validate the possibility of using small spectrum-width optical source in the optical transmitter not only for optical access but also for long haul optical transmission. However a dramatic reduction in the transmission distance occurs when the bit rate reaches 10 Gbps as shown in Figure 4. In this case, only 11.56 km fiber length can be used if 1dB penalty is permitted, which strongly necessitates incorporation of a dispersion compensation technique in the transmission line.

IV. CONCLUSIONS

The maximum optical transmission length has been estimated under the effect of GVD when an optical source with large spectral width and an optical source with small spectral width were assumed, respectively. Bit rates of 622 Mbps, 1 Gps, 2.5 Gbps, and 10 Gbps were used in this study as they are the most commonly used in the optical access systems. Although the results show a significant improvement in case of using an optical source with small spectral width in which the GVD noise floor is reached at much longer fiber, a dramatic reduction in the transmission length occurs when the bit rate reaches 10 Gbps, which necessitates incorporation of a dispersion compensation technique in the link.

REFERENCES


